Aggregation of Demand-Side Flexibility in Electricity Markets: Negative Impact Analysis and Mitigation Method

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***Abstract*—Aggregation of demand-side flexibility plays a crucial role in helping improve the system-wide performance of power grids. However, little considered is the potential negative impact of self-interested flexibility aggregators, who are being strategic for their own benefit at the cost of other market participants or even system-wide performance. This article aims to theoretically analyze this negative impact, as well as propose a corresponding mitigation method. Specifically, we consider a strategic aggregator that derives the optimal bidding strategy of the flexibility bounds (for cumulative energy and instantaneous power consumption) and trades electricity in a pool. A multi-period bi-level program with a DC network setup is considered. The upper-level problem represents the aggregator's cost minimization, and the lower-level problem represents the market clearing process. Based on this bi-level formulation, our theoretical analysis shows that the potential negative impact of the strategic behavior on the system generation cost, the payment of the fixed loads, and the payment of the non-strategic aggregators depends on the bus locations of both the strategic and non-strategic aggregators. We propose to additionally charge the strategic aggregator for the newly introduced congestion so as to avoid system performance degradation. The analytical results are validated via simulations.**

***Index Terms*—Demand-side flexibility, strategic bidding, bi-level programming, bus locations.**

I. Introduction

ELASTIC demand, such as thermostatically controlled loads and electric vehicle (EV) charging loads, can provide an enormous amount of demand-side flexibility (DSF) to the power grid. With appropriately designed economic and incentive mechanisms, DSF can improve the reliability and efficiency of power system operation [1]. For instance, as a prominent example of flexible loads in power grids, the charging demand of EVs can be flexibly adjusted in multiple dimensions, including total energy demand, power rating, charging duration, charging location, etc. This process is referred to as multi-dimensional DSF [2], [3]. In recent years, many papers have investigated aspects of integrating flexible EV charging loads, such as coordinated EV charging control for peak-shaving [4], frequency regulation [5], and operating reserves [6], into the power grid.

In practice, many small-scale flexible loads are geographically distributed in decentralized control areas. Thus, aggregators play a pivotal role exploiting the system value of DSF. Through the aggregation process, groups of small-scale flexible loads can merge their consumption requirements in grid-scale, and this enables them to participate in the bidding process of electricity markets [7]. Furthermore, a load aggregator can serve as an interface between the grid operator, i.e., the independent system operator (ISO), and the end users, with a joint consideration of both sides. Therefore, it is possible that both the global objective of the system and the local objectives of end users can be optimized simultaneously, resulting in an appealing win-win solution [8], [9].

However, the participation of aggregators in electricity markets, though providing the aforementioned benefits, also introduces unique challenges to the system operator due to their potential to manipulate prices. Specifically, instead of the flexible region being in line with the preferences of the aggregated flexible loads, the aggregator can strategically design its flexible region based on its knowledge of the network topology. Since the ISO clears the market according to the bids of the flexible region submitted by the aggregator, the aggregator has the opportunity to manipulate prices and therefore may have a significant impact on the system performance if its market share is large. Thus, understanding the potential of these aggregators to deteriorate the system performance is of great importance so that regulatory authorities can take appropriate steps to mitigate their negative impacts as needed.

A. Related Work

Elastic load aggregators are becoming proactive participants in electricity markets instead of simply followers, and the literature examines their role from a number of perspectives. On the one hand, many works discuss managing DSF in electricity markets via demand response from the system point of view [2], [3]. All these related works demonstrate the positive effects brought by DSF on the system performance,

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e.g., decreasing the total system cost, increasing the social welfare, and flattening the locational marginal prices (LMPs) over time. On the other hand, many works consider the aggregators' perspective to study how demand aggregators derive optimal bidding strategies to alter pool prices for their own benefit [10], [11], [12]. It is, therefore, natural to question whether these strategic behaviors could have negative effects on the system or other market participants, which is the focus of this article.

In fact, with the liberalization of electricity markets, the price manipulation of the market participants has become a significant issue. Many existing works have studied the strategic producers in networked settings. For example, [9], [13], and [14] use a bi-level game, conjectured supply function model, and Cournot competition, respectively, to study oligopoly competition. The results in [15] and [16] show that the strategic behaviors of the market participants may indeed have negative impacts on the social welfare and lead to system-wide efficiency losses. Reference [17] conducts a survey on identifying and measuring market power for conventional generators in an electricity market. More recently, [18] offers insight into market manipulation by renewable generation producers via strategic curtailment.

Potential negative impacts have also been observed from strategic flexible load aggregators. For example, the numerical results in [11] show that the market power of the strategic consumer is enhanced in proportion to its reserve capacity, and can increase the required down reserve in the balancing stage through significantly impacting day-ahead wind scheduling. Moreover, in current practice within the European power market setup, the maximum bid size limits of "balance responsible parties", i.e., some form of demand aggregators, can restrain their market power to some extent and enable the market functioning [19]. However, none of the existing works have conducted a deep theoretical analysis of the problem. To be specific, for the system operators, it is still not clear what negative effects strategic aggregators might bring, how to identify which aggregator(s) is a potential troublemaker, nor how the price manipulation happens. More research is needed to provide insights for improving the market design, apart from simply limiting the size. In this regard, this article is trying to push forward the frontier.

In this article, we demonstrate that the degraded system performances caused by strategic DSF aggregators include higher system generation cost, higher payment of the fixed loads, and higher payment of the non-strategic DSF aggregators. In our settings of transmission-level DC power networks, we find that the potential troublemakers are closely related to their bus locations, i.e., the shift factors. We theoretically derive the conditions for degraded system performances related to the shift factors of all different market participants. We also demonstrate that certain strategic DSF aggregators can save costs by causing additional line congestion. Similar phenomena have also been reported by literature in other contexts. For example, [20] demonstrates that load flexibility brings economic benefits to the market participants through enabling congestion-free dispatch. Reference [21] shows that cyber attacks can impact electricity prices by causing line congestion, and [22] shows that the system efficiency is not guaranteed when congestion occurs. However, none of these works provided theoretical evidence of these phenomena as we do.

B. Summary of Contributions

This article provides an algorithmic framework for investigating the participation of a strategic DSF aggregator in electricity markets. We focus on transmission networks with DC approximation. A bi-level model of an aggregator is considered, where the upper level is the aggregator's cost minimization problem that decides the optimal bidding of the flexible region, and the lower level is the ISO's generation cost minimization problem that decides the resources dispatch and the LMPs. Similar bi-level models have been widely adopted in the recent literature studying the strategic behaviors of the participants in electricity markets [18], [23], [24], [25]. This bi-level problem is essentially a mathematical problem with equilibrium constraints (MPEC) [26]. It can be reformulated as an equivalent mixed integer convex program by applying the Big-M method [27] and the Karush–Kuhn–Tucker (KKT) optimality conditions. Based on this framework, we first analyze the impact of strategic aggregators in deteriorating the system performance, and we then propose a mitigation method. Our two main contributions are summarized as follows.

First, we examine the aggregators' opportunities for price manipulation via strategic bidding in the transmission-level electricity market. Our results reveal that the strategic bidding can save the strategic aggregators from much higher payments and can have a significant impact on the system performance, including the generation cost, the payment of the fixed loads, and the payment of the non-strategic aggregators. In particular, the bus locations of the strategic and non-strategic aggregators play an important role. Under different relations between the shift factor values of all market participants, we theoretically derive the different impacts on the system performance caused by the strategic bidding. This means that the potential negative impacts of the strategic aggregators can be anticipated beforehand based only on the network topology and parameters. This could provide the system operator with insights into the necessary actions to mitigate these negative impacts.

Second, we propose a tariff scheme as a possible solution used for mitigating the negative impacts on system performance induced by the strategic aggregators. According to the theoretical analysis of the bi-level model, we find that the system performance deterioration is closely related to the newly introduced congestion resulting from the strategic bidding. Specifically, the increased generation cost and increased payment of the fixed load always occur simultaneously with the increased congestion. Therefore, we propose that the ISO collects penalty charges for the newly introduced congestion from the aggregators, and uses these penalty charges to compensate the other participants who suffer from increasing payments due to the newly created congestion. It can be shown, both theoretically and numerically, that our proposed tariff scheme can prevent the strategic aggregators from increasing the system generation costs and the payments of other participants.

II. System Model

In this section, the power system model is defined. We focus on the day-ahead economic dispatch on the transmission level over a fixed time horizon $\mathcal{T} = \{1, 2, \ldots, T\}$. We consider a network model with the set of buses $\mathcal{N} = \{1, 2, \ldots, N\}$ and the set of transmission lines $\mathcal{L} = \{1, 2, \ldots, L\}$. The generation and fixed load at bus $n \in \mathcal{N}$ at time $t \in \mathcal{T}$ are respectively denoted by g_{tn} and d_{tn}, with $\mathbf{g}_{t\cdot} = [g_{t1}, \ldots, g_{tN}]$, $\mathbf{g}_{\cdot n} = [g_{1n}, \ldots, g_{Tn}]^T$ and $\mathbf{d}_{t\cdot} = [d_{t1}, \ldots, d_{tN}]$, $\mathbf{d}_{\cdot n} = [d_{1n}, \ldots, d_{Tn}]^T$.

Besides the fixed loads, we consider multiple DSF aggregators, which aggregate a large amount of small-scale flexible loads such as EVs. The aggregated flexible load at bus n at time t is denoted by x_{tn} with $\mathbf{x}_{t\cdot} = [x_{t1}, \ldots, x_{tN}]$ and $\mathbf{x}_{\cdot n} = [x_{1n}, \ldots, x_{Tn}]^T$. Specifically, two dimensions of flexibility are considered, namely, energy-flexibility and power-flexibility. In the day-ahead market, the DSF aggregators need to bid a set of flexibility bounds to the ISO. The bidding set of the aggregator at bus n at time t is $\{l_{tn}, u_{tn}, x_{tn}^-, x_{tn}^+\}$, which includes the lower bound l_{tn} and upper bound u_{tn} for the cumulative energy consumption, and the lower bound x_{tn}^- and upper bound x_{tn}^+ for the instantaneous power consumption. Note that $\mathbf{g}_{\cdot n}$, $\mathbf{d}_{\cdot n}$, or $\mathbf{x}_{\cdot n}$ can be set to $\mathbf{0}$, a $T \times 1$ zero vector, when there is no generator, fixed load, or flexible load located at bus n, respectively.

A. Model of ISO's Market Clearing

After collecting the bids, the ISO solves a constrained optimization problem for market clearing and determines the generation $\{g_{tn}\}_{\forall t \in \mathcal{T}, n \in \mathcal{N}}$ and flexible load consumption $\{x_{tn}\}_{\forall t \in \mathcal{T}, n \in \mathcal{N}}$. The objective of the optimization is to minimize the total generation cost based on the network information. The LMPs are announced as a function of the optimal Lagrange multipliers of this optimization problem. Mathematically, the following program has to be solved:

$$\min_{\boldsymbol{g}, \boldsymbol{x}} \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \mathcal{C}_n^G(g_{tn}) \tag{1a}$$

$$\text{s.t.} \quad \lambda_t : \big(\boldsymbol{g}_{t\cdot} - \boldsymbol{d}_{t\cdot} - \boldsymbol{x}_{t\cdot}\big)\mathbf{1} = 0, \forall t \in \mathcal{T}, \tag{1b}$$

$$\boldsymbol{\mu}_{t\cdot}^-, \boldsymbol{\mu}_{t\cdot}^+ : -\boldsymbol{c} \leq \boldsymbol{H}\big(\boldsymbol{g}_{t\cdot} - \boldsymbol{d}_{t\cdot} - \boldsymbol{x}_{t\cdot}\big)^T \leq \boldsymbol{c}, \forall t \in \mathcal{T}, \tag{1c}$$

$$\boldsymbol{\gamma}_{\cdot n}^{g-}, \boldsymbol{\gamma}_{\cdot n}^{g+} : \boldsymbol{g}_{\cdot n}^- \leq \boldsymbol{g}_{\cdot n} \leq \boldsymbol{g}_{\cdot n}^+, \forall n \in \mathcal{N}, \tag{1d}$$

$$\boldsymbol{\gamma}_{\cdot n}^{e-}, \boldsymbol{\gamma}_{\cdot n}^{e+} : \boldsymbol{l}_{\cdot n} \leq \boldsymbol{L}\boldsymbol{x}_{\cdot n} \leq \boldsymbol{u}_{\cdot n}, \forall n \in \mathcal{N}, \tag{1e}$$

$$\boldsymbol{\gamma}_{\cdot n}^{p-}, \boldsymbol{\gamma}_{\cdot n}^{p+} : \boldsymbol{x}_{\cdot n}^- \leq \boldsymbol{x}_{\cdot n} \leq \boldsymbol{x}_{\cdot n}^+, \forall n \in \mathcal{N}, \tag{1f}$$

where the generation cost $\mathcal{C}_n^G(\cdot)$ in the objective is assumed to be a convex function. Constraints (1b) represent the power balance equations and (1c) denote the transmission line capacity constraints, where $\mathbf{c} = [c_1, \ldots, c_L]^T$ denotes the line capacities and the element H_{ln} of the matrix $\boldsymbol{H} \in \mathbb{R}^{L \times N}$ denotes the shift factor, known as the power transfer distribution factor (PTDF), of line l with respect to bus n. Constraints (1d), (1e), and (1f) are the generation capacity constraints, cumulative energy constraints, and instantaneous power constraints of the DSF aggregators, respectively, where $\boldsymbol{L} \in \mathbb{R}^{T \times T}$ is a lower triangular matrix with all non-zero elements equal to one serving for calculating the cumulative energy consumption. The dual variables of constraints (1b)–(1f) are denoted by $\{\boldsymbol{\lambda}; \boldsymbol{\mu}^-, \boldsymbol{\mu}^+; \boldsymbol{\gamma}^{g-}, \boldsymbol{\gamma}^{g+}; \boldsymbol{\gamma}^{e-}, \boldsymbol{\gamma}^{e+}; \boldsymbol{\gamma}^{p-}, \boldsymbol{\gamma}^{p+}\}$, respectively, where $\boldsymbol{\lambda} \in \mathbb{R}^{T \times 1}$, $\{\boldsymbol{\mu}^-, \boldsymbol{\mu}^+\} \in \mathbb{R}^{T \times L}$ and the dimensions of the remainder are $T \times N$. The LMP matrix $\boldsymbol{\pi} \in \mathbb{R}^{T \times N}$ is calculated by $\boldsymbol{\pi} = \boldsymbol{\lambda}\mathbf{1}^T + (\boldsymbol{\mu}^- - \boldsymbol{\mu}^+)\boldsymbol{H}$, where $\mathbf{1}$ is an $N \times 1$ all-ones vector. Each element π_{tn} denotes the LMP for bus n at time t.

B. Model of Strategic DSF Aggregators

Recall that we focus on the strategic behaviors of DSF aggregators. Traditionally, the DSF aggregator determines its bids, i.e., the aggregated flexible region, based on the collection of all its aggregated flexible loads' consumption requirements. We refer to this as the preferred flexible region in this article. Taking an EV as an example, its own preferred flexible region is based on its actual traveling requirements, e.g., at what time slots the battery should be charged, to what level intervals, and at what rate intervals. However, if the DSF aggregator has knowledge about the network, it will submit strategic bids which deviate from the original preferred flexible region as long as there is a chance to achieve a lower energy payment. After being informed of the power dispatch from the ISO, the aggregator has to distribute this aggregated power dispatch to all its aggregated EVs. Note that for each EV, the deviation of the power dispatch from its preferred region is acceptable once a reasonable pricing scheme for compensation is established. The detailed distribution scheme and the corresponding pricing scheme between the aggregator and its aggregated flexible loads are beyond the scope of our discussion. The problem to determine the best bidding strategy for DSF aggregator i (located at bus i) is

$$\min_{\boldsymbol{x}_{\cdot i}, \boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}_{\cdot i}^-, \boldsymbol{x}_{\cdot i}^+} \mathcal{C}_i^U\big(\boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}_{\cdot i}^-, \boldsymbol{x}_{\cdot i}^+\big) + \boldsymbol{\pi}_{\cdot i}^T \boldsymbol{x}_{\cdot i} \tag{2a}$$

$$\text{s.t.} \quad \boldsymbol{l}_{\cdot i} \leq \boldsymbol{L}\boldsymbol{x}_{\cdot i} \leq \boldsymbol{u}_{\cdot i}, \tag{2b}$$

$$\boldsymbol{x}_{\cdot i}^- \leq \boldsymbol{x}_{\cdot i} \leq \boldsymbol{x}_{\cdot i}^+, \tag{2c}$$

$$\boldsymbol{l}_{\cdot i} \geq \underline{\boldsymbol{l}}_{\cdot i}, \tag{2d}$$

$$\boldsymbol{u}_{\cdot i} \leq \overline{\boldsymbol{u}}_{\cdot i}, \tag{2e}$$

$$\boldsymbol{x}_{\cdot i}^- \geq \underline{\boldsymbol{x}}_{\cdot i}^-, \tag{2f}$$

$$\boldsymbol{x}_{\cdot i}^+ \leq \overline{\boldsymbol{x}}_{\cdot i}^+. \tag{2g}$$

The first term in the objective (2a) is a convex utility cost function incurred by the deviation from the *preferred flexibility bounds* $\{\hat{\boldsymbol{l}}_{\cdot i}, \hat{\boldsymbol{u}}_{\cdot i}, \hat{\boldsymbol{x}}_{\cdot i}^-, \hat{\boldsymbol{x}}_{\cdot i}^+\}$, which should be submitted truthfully if there is no strategic behavior. Specifically, we assume the utility cost has the following linear form: $\mathcal{C}_i^U(\boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}_{\cdot i}^-, \boldsymbol{x}_{\cdot i}^+) = a_i(\|\boldsymbol{l}_{\cdot i} - \hat{\boldsymbol{l}}_{\cdot i}\|_1 + \|\boldsymbol{u}_{\cdot i} - \hat{\boldsymbol{u}}_{\cdot i}\|_1 + \|\boldsymbol{x}_{\cdot i}^- - \hat{\boldsymbol{x}}_{\cdot i}^-\|_1 + \|\boldsymbol{x}_{\cdot i}^+ - \hat{\boldsymbol{x}}_{\cdot i}^+\|_1)$. Here a_i is the deviation coefficient predetermined by the strategic aggregator, to weight the deviation from the preferred flexibility bounds. The second term in (2a) is the energy payment over the entire time horizon. Constraints (2b) and (2c) are the cumulative energy constraints and instantaneous power constraints for the aggregated flexible loads. Constraints (2d)–(2g) are the physical limits for the bids based on the *physical flexibility bounds* $\{\underline{\boldsymbol{l}}_{\cdot i}, \overline{\boldsymbol{u}}_{\cdot i}, \underline{\boldsymbol{x}}_{\cdot i}^-, \overline{\boldsymbol{x}}_{\cdot i}^+\}$. For EVs, their physical flexibility bounds refer to the technically achievable battery charging limits. Among the optimal decisions, a subset containing all the *optimal flexibility bounds* $\{\boldsymbol{l}^*_{\cdot i}, \boldsymbol{u}^*_{\cdot i}, \boldsymbol{x}^{-*}_{\cdot i}, \boldsymbol{x}^{+*}_{\cdot i}\}$ will be used as bids submitted to the ISO. However, since the real dispatch of the aggregated flexible loads' consumption will be determined by the ISO's market clearing, the optimal decision $\boldsymbol{x}^*_{\cdot i}$ will not be adopted for dispatch.

C. Bi-Level to Single-Level Reformulation

We can observe that in the local optimization problem of the strategic aggregator, the LMP vector $\pi_{\cdot i}$ is the clearing result of the ISO's economic dispatch. Therefore, the aggregator's problem is a bi-level optimization problem. The upper-level problem is the local optimization in Eq.(2). The lower-level problem, i.e., the wholesale market clearing in Eq.(1), is considered to estimate the LMPs. The bi-level problem can be reformulated into a single-level problem as follows:

$$\min_{\boldsymbol{x}_{\cdot i}, \boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}^-_{\cdot i}, \boldsymbol{x}^+_{\cdot i}} \mathcal{C}^U_i\left(\boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}^-_{\cdot i}, \boldsymbol{x}^+_{\cdot i}\right) + \boldsymbol{\pi}^T_{\cdot i}\boldsymbol{x}_{\cdot i} \tag{3a}$$

$$s.t. \quad \text{Constraints (2b)–(2g)},$$

$$\boldsymbol{\pi} = \boldsymbol{\lambda}\mathbf{1}^T + \left(\boldsymbol{\mu}^- - \boldsymbol{\mu}^+\right)\boldsymbol{H}, \tag{3b}$$

$$\left\{\boldsymbol{\lambda}, \boldsymbol{\mu}^+, \boldsymbol{\mu}^-\right\} \text{ solve (1)}. \tag{3c}$$

The lower-level problem can be included into the upper-level problem based on its equivalent KKT conditions since it is continuous and convex. Then we end up with an MPEC. In Appendix A, the detailed formulation and solution method of the MPEC is provided. In view of the non-convexities in terms of both the complementary conditions and the bilinear term $\boldsymbol{\pi}^T_{\cdot i}\boldsymbol{x}_{\cdot i}$ in the objective function, the Big-M method [27] and the derivation based on the KKT conditions are separately applied for the linearizations.

Note that an important assumption about the reformulation (3) is that the aggregator has complete knowledge of the generation cost function $\mathcal{C}^G(\cdot)$, the network topology, i.e., the shift factor matrix $\boldsymbol{H}$ and the line capacities $\boldsymbol{c}$, the state estimates of fixed loads $\boldsymbol{d}$, and the bids of other aggregators $\{\boldsymbol{l}_{\cdot n}, \boldsymbol{u}_{\cdot n}, \boldsymbol{x}^-_{\cdot n}, \boldsymbol{x}^+_{\cdot n}\}_{\forall n \neq i}$. This assumption may be over-optimistic, but an ideal market design is that aggregators do not have profitable manipulations at the expense of increasing the system generation cost, even with such full knowledge [18]. The results in this article indicate that the current market does not conform to this ideal market design.

Simply solving the single-level reformulation (3) numerically cannot lead to any structural insight into the accommodation of strategic DSF aggregators in electricity markets in practice. This motivates our analysis in the next section.

III. Impact Analysis and Mitigation Method

In this section, we present our main results. Specifically, we theoretically analyze the strategic aggregator's impacts on the system. We focus on three aspects of the system performance, namely, the system-wide generation cost, the payment of fixed loads, and the payment of non-strategic DSF aggregators. The system performance is compared with the benchmark case where the strategic DSF aggregator submits its preferred flexibility bounds directly, without solving for any strategic bids. Below, we first present the details of the benchmark case with some technical assumptions to aid our theoretic analysis.

A. Benchmark and Assumptions

In the benchmark case, the ISO clears the market by solving the following problem (4) based on the preferred flexibility bounds of aggregator i, as shown in the constraints (4g) and (4h). The optimal solutions are denoted with superscript 0.

$$\min_{\boldsymbol{g}, \boldsymbol{x}} \sum_{t \in \mathcal{T}} \mathcal{C}^G\left(\boldsymbol{g}_{t\cdot}\right) \tag{4a}$$

$$s.t. \quad \lambda_t : \left(\boldsymbol{g}_{t\cdot} - \boldsymbol{d}_{t\cdot} - \boldsymbol{x}_{t\cdot}\right)\mathbf{1} = 0, \forall t \in \mathcal{T}, \tag{4b}$$

$$\boldsymbol{\mu}^-_{t\cdot}, \boldsymbol{\mu}^+_{t\cdot} : -\boldsymbol{c} \leq \boldsymbol{H}\left(\boldsymbol{g}_{t\cdot} - \boldsymbol{d}_{t\cdot} - \boldsymbol{x}_{t\cdot}\right)^T \leq \boldsymbol{c}, \forall t \in \mathcal{T}, \tag{4c}$$

$$\boldsymbol{\gamma}^{g-}_{\cdot n}, \boldsymbol{\gamma}^{g+}_{\cdot n} : \boldsymbol{g}^-_{\cdot n} \leq \boldsymbol{g}_{\cdot n} \leq \boldsymbol{g}^+_{\cdot n}, \forall n \in \mathcal{N}, \tag{4d}$$

$$\boldsymbol{\gamma}^{e-}_{\cdot n}, \boldsymbol{\gamma}^{e+}_{\cdot n} : \boldsymbol{l}_{\cdot n} \leq \boldsymbol{L}\boldsymbol{x}_{\cdot n} \leq \boldsymbol{u}_{\cdot n}, \forall n \neq i, \tag{4e}$$

$$\boldsymbol{\gamma}^{p-}_{\cdot n}, \boldsymbol{\gamma}^{p+}_{\cdot n} : \boldsymbol{x}^-_{\cdot n} \leq \boldsymbol{x}_{\cdot n} \leq \boldsymbol{x}^+_{\cdot n}, \forall n \neq i, \tag{4f}$$

$$\boldsymbol{\gamma}^{e-}_{\cdot i}, \boldsymbol{\gamma}^{e+}_{\cdot i} : \hat{\boldsymbol{l}}_{\cdot i} \leq \boldsymbol{L}\boldsymbol{x}_{\cdot i} \leq \hat{\boldsymbol{u}}_{\cdot i}, \tag{4g}$$

$$\boldsymbol{\gamma}^{p-}_{\cdot i}, \boldsymbol{\gamma}^{p+}_{\cdot i} : \hat{\boldsymbol{x}}^-_{\cdot i} \leq \boldsymbol{x}_{\cdot i} \leq \hat{\boldsymbol{x}}^+_{\cdot i}. \tag{4h}$$

Given the bi-level form of the strategic aggregator's optimization, it is challenging to theoretically characterize the effect of the optimal bidding on the system performance. However, it turns out that under certain simplifications, the impact of the strategic bidding can indeed be captured by the bus location of the strategic DSF aggregator. In particular, the discussions in this section are based on the following assumptions.

1) There is one generator at bus g, one strategic DSF aggregator at bus i, one non-strategic aggregator at bus j, and multiple fixed loads at buses $n \in \mathcal{N}$. It is nontrivial to analyze the cases that any of the numbers of the generators, strategic aggregators, or non-strategic aggregators exceeds one.

2) The generation cost function is quadratic as $\mathcal{C}^G(g_t) = Ag_t^2 + Bg_t + C$, where A, B, and C are nonnegative parameters.

3) The total load profile over the entire network is dominated by the fixed loads ($\sum_n d_{tn} \gg \max\{x_{ti}, x_{tj}\}, \forall t$).

4) The entire time horizon $\mathcal{T}$ is divided into two periods, $\mathcal{T}_c$ and $\mathcal{T}_{uc}$. Congestion may occur in $\mathcal{T}_c$, while it never occurs in $\mathcal{T}_{uc}$. At time $t \in \mathcal{T}_{uc}$, the fixed load profile is comparatively flat around the average value $\overline{d}_n = \frac{1}{|\mathcal{T}_{uc}|}\sum_{t \in \mathcal{T}_{uc}} d_{tn}$. Similarly, the generation is around $\overline{g} = \frac{1}{|\mathcal{T}_{uc}|}\sum_{t \in \mathcal{T}_{uc}} g_t$. At time $t \in \mathcal{T}_c$, the fixed load is heavy ($d_{tn} \gg \overline{d}_n$) and congestion may occur.

5) The cumulative energy consumption of flexible loads over the entire time horizon is fixed, i.e., $\sum_t x_{ti} = X_i$ and $\sum_t x_{tj} = X_j$ (X_i and X_j are constant parameters).

B. Impact Analysis: Structural Properties

The impacts of the strategic DSF aggregator on the system-wide performance are summarized in Theorem 1. We use the term *win* to refer to the lower total cost for the strategic DSF aggregator, lower generation cost for the system, lower payment for the non-strategic DSF aggregator, and lower payment for the fixed load, compared with the benchmark.

Denote the set of possible congested lines by $\mathcal{L}_c$. We have $H_{ln} < \min\{H_{lg}, H_{li}, H_{lj}\}, \forall l \in \mathcal{L}_c$, due to the dominance of the fixed loads. Note that the relative differences in the shift factor values of the same line are independent of the slack bus. The line direction is defined to let the constraint associated with $c_l (> 0)$, instead of $-c_l$, be binding. Thus, we only need to tackle $\mu_{tl}^{+} > 0$ when congestion occurs. We assume the generation output has not reached the limits, i.e., $\boldsymbol{g}^{-} < \boldsymbol{g} < \boldsymbol{g}^{+}$ and $\boldsymbol{\gamma}^{g-} = \boldsymbol{\gamma}^{g+} = 0$. Suppose an individual fixed load is proportional to the total amount of fixed loads, i.e., $d_{tn} = p_n \sum_n d_{tn}, \forall t$, where p_n is a scaling factor with $\sum_n p_n = 1$. Let $\Delta x_{ti} = x_{ti}^{*} - x_{ti}^{0}$, $\Delta x_{tj} = x_{tj}^{*} - x_{tj}^{0}$, and $\Delta g_t = g_t^{*} - g_t^{0}$, $t \in \mathcal{T}_c$. Based on our settings, it is easy to see that the aggregator will make the same direction of strategic changes during the congestion period, i.e., $\{\Delta x_{ti}\}_{\forall t \in \mathcal{T}_c}$ have the same sign.

Theorem 1: Albeit surely benefiting the strategic DSF aggregator, the impacts of its strategic behavior on the system performance depend on the bus locations, i.e., the shift factors, of both the strategic and non-strategic DSF aggregators.

- *Scenario-I (Quadruple-win):* When $H_{li} < H_{lg}$, $H_{lj} < H_{lg}$, $\forall l \in \mathcal{L}_c$, the system generation cost and the payments of the fixed load and the non-strategic DSF aggregator are lower than those under the benchmark case.
- *Scenario-II (Triple-win):* When $H_{li} < H_{lg}$, $H_{lj} > H_{lg}$, $\forall l \in \mathcal{L}_c$, the system generation cost and the payment of the fixed load are lower than those under the benchmark case.
- *Scenario-III (Double-win):* When $H_{li} > H_{lg}$, $H_{lj} < H_{lg}$, $\forall l \in \mathcal{L}_c$, the system generation cost is lower than that under the benchmark case.
- *Scenario-IV (Single-win):* When $H_{li} > H_{lg}$, $H_{lj} > H_{lg}$, $\forall l \in \mathcal{L}_c$, the system generation cost is higher than that under the benchmark case once one of the following conditions is satisfied: $H_{li} > H_{lj}$ and $\sum_{t \in \mathcal{T}_c} \Delta x_{ti} < -\frac{\sum_{t \in \mathcal{T}_c}(c_l - f_{tl}^{0})}{H_{li} - H_{lj}}$, $l \in \mathcal{L}_c$; $H_{li} < H_{lj}$ and $\sum_{t \in \mathcal{T}_c} \Delta x_{ti} > \frac{\sum_{t \in \mathcal{T}_c}(c_l - f_{tl}^{0})}{H_{lj} - H_{li}}$, $l \in \mathcal{L}_c$.

Proof: The changes of the strategic aggregator's payment, ΔP_i, the payment of the fixed load at bus n, ΔP_n, and the power flow at line l at time t, ΔFlow_{tl}, are given by (5), (6), and (7), respectively. Please refer to Appendix B for the detailed derivations.

$$\Delta P_i = \sum_{t \in \mathcal{T}_c} \left[2A\left(g_t^{*} - \overline{g}\right)\Delta x_{ti} + \sum_{l \in \mathcal{L}_c} \left(\mu_{tl}^{*} x_{ti}^{*} - \mu_{tl}^{0} x_{ti}^{0}\right)\left(H_{lg} - H_{li}\right) \right] \quad (5)$$

$$\Delta P_n = \sum_{t \in \mathcal{T}_c} \left[2A\left(d_{tn} - \overline{d}_n\right)\Delta g_t + \sum_{l \in \mathcal{L}_c} \left(\mu_{tl}^{*} - \mu_{tl}^{0}\right) d_{tn}\left(H_{lg} - H_{ln}\right) \right] \quad (6)$$

$$\Delta \text{Flow}_{tl} = \left(H_{lg} - H_{li}\right)\Delta x_{ti} + \left(H_{lg} - H_{lj}\right)\Delta x_{tj}. \quad (7)$$

Next, we discuss all possible scenarios.

Scenario-I ($H_{li} < H_{lg}$, $H_{lj} < H_{lg}$):

Sub-scenario 1: if $\Delta x_{ti} > 0$, then $2A(g_t^{*} - \overline{g})\Delta x_{ti} > 0$ and $x_{ti}^{*} > x_{ti}^{0}$, $t \in \mathcal{T}_c$. Thus, we have $\mu_{tl}^{+*} x_{ti}^{*} - \mu_{tl}^{+0} x_{ti}^{0} < 0$ and $\mu_{tl}^{+*} \le \mu_{tl}^{+0}$. Sub-scenario 2: if $\Delta x_{ti} < 0$, then $2A(g_t^{*} - \overline{g})\Delta x_{ti} < 0$ and $x_{ti}^{*} < x_{ti}^{0}$, $t \in \mathcal{T}_c$. Thus, we have $\mu_{tl}^{+*} x_{ti}^{*} - \mu_{tl}^{+0} x_{ti}^{0} < 0$ and $\mu_{tl}^{+*} \le \mu_{tl}^{+0}$. Apparently, sub-scenario 2 will save the largest payment amount with both terms in ΔP_i negative. Therefore, sub-scenario 1 will never occur. When $H_{lj} < H_{lg}$, it must hold that the consumption of the non-strategic aggregator is already as high as possible during the peak hours and reach the upper flexibility bounds, so $\Delta x_{tj} = 0$. As a result, $\Delta g_t = \Delta x_{ti} + \Delta x_{tj} = \Delta x_{ti} < 0$ and the generation cost will decrease. Since $\Delta P_n = \sum_{t \in \mathcal{T}_c} [2A(d_{tn} - \overline{d}_n)\Delta g_t + \sum_{l \in \mathcal{L}_c} (\mu_{tl}^{*} - \mu_{tl}^{0}) d_{tn}(H_{lg} - H_{ln})] < 0$, the payment of the fixed load will decrease. Since $\Delta P_j = \sum_{t \in \mathcal{T}_c} [2A(g_t^{*} - \overline{g})\Delta x_{tj} + \sum_{l \in \mathcal{L}_c} (\mu_{tl}^{*} x_{tj}^{*} - \mu_{tl}^{0} x_{tj}^{0})(H_{lg} - H_{lj})] < 0$, the payment of the non-strategic aggregator will decrease as well.

Scenario-II ($H_{li} < H_{lg}$, $H_{lj} > H_{lg}$, $\forall l \in \mathcal{L}_c$): Different from the Quadruple-win scenario, when $H_{lj} > H_{lg}$, it must hold that $\Delta x_{tj} \le 0$, $t \in \mathcal{T}_c$. Thus, $\Delta g_t = \Delta x_{ti} + \Delta x_{tj} \le \Delta x_{ti} < 0$ and the generation cost will decrease. Since $\Delta P_n = \sum_{t \in \mathcal{T}_c} [2A(d_{tn} - \overline{d}_n)\Delta g_t + \sum_{l \in \mathcal{L}_c} (\mu_{tl}^{*} - \mu_{tl}^{0}) d_{tn}(H_{lg} - H_{ln})] < 0$, the payment of the fixed load will decrease as well.

Scenario-III ($H_{li} > H_{lg}$, $H_{lj} < H_{lg}$, $\forall l \in \mathcal{L}_c$): When $H_{lj} < H_{lg}$, it must hold that the consumption of the non-strategic aggregator is already as high as possible during the peak hours reaching the upper flexibility bounds, so $\Delta x_{tj} = 0$, $t \in \mathcal{T}_c$. Sub-scenario 1: if $\Delta x_{ti} > 0$, then $2A(g_t^{*} - \overline{g})\Delta x_{ti} > 0$ and $x_{ti}^{*} > x_{ti}^{0}$. Thus, we have $\mu_{tl}^{+*} = 0$. As a result, both terms in ΔP_i are positive and this sub-scenario will never occur. Sub-scenario 2: if $\Delta x_{ti} < 0$, then $2A(g_t^{*} - \overline{g})\Delta x_{ti} < 0$ and $x_{ti}^{*} < x_{ti}^{0}$. Thus, we have $\mu_{tl}^{+*} > \mu_{tl}^{+0}$ and $\mu_{tl}^{+*} x_{ti}^{*} - \mu_{tl}^{+0} x_{ti}^{0} > 0$. As a result, $\Delta g_t = \Delta x_{ti} + \Delta x_{tj} = \Delta x_{ti} < 0$ and the generation cost will decrease.

Scenario-IV ($H_{li} > H_{lg}$, $H_{lj} > H_{lg}$, $\forall l \in \mathcal{L}_c$):

Sub-scenario 1: if $\Delta x_{ti} > 0$, then $2A(g_t^{*} - \overline{g})\Delta x_{ti} > 0$ and $x_{ti}^{*} > x_{ti}^{0}$. Thus, we have $\mu_{tl}^{+*} x_{ti}^{*} - \mu_{tl}^{+0} x_{ti}^{0} > 0$ and $\mu_{tl}^{+*} > \mu_{tl}^{+0}$, $t \in \mathcal{T}_c$. As a result, $\Delta x_{tj} < 0$. It is possible that $\Delta g = \sum_{t \in \mathcal{T}_c} \Delta g_t = \sum_{t \in \mathcal{T}_c} (\Delta x_{ti} + \Delta x_{tj}) > 0$, which means the generation cost will increase. Sub-scenario 2: if $\Delta x_{ti} < 0$, then $2A(g_t^{*} - \overline{g})\Delta x_{ti} < 0$ and $x_{ti}^{*} < x_{ti}^{0}$. Thus, we have $\mu_{tl}^{+*} x_{ti}^{*} - \mu_{tl}^{+0} x_{ti}^{0} > 0$ and $\mu_{tl}^{+*} > \mu_{tl}^{+0}$. As a result, $\Delta x_{tj} \ge 0$. It is possible that $\Delta g = \sum_{t \in \mathcal{T}_c} \Delta g_t = \sum_{t \in \mathcal{T}_c} (\Delta x_{ti} + \Delta x_{tj}) > 0$, which means the generation cost will increase. Based on the equation $\sum_{t \in \mathcal{T}_c} (\Delta x_{ti} + \Delta x_{tj}) > 0$, we can derive the conditions in the theorem by replacing Δx_{tj} with Δx_{ti} using the relationship in (7). ■

The key phenomenon revealed by Theorem 1 can be explained in an intuitive way: Once the strategic aggregator is located at a bus with a comparatively large shift factor with respect to the congested line, it can manipulate its consumption curve to introduce more congestion and therefore decrease its nodal prices, which can save on its energy payment as a result. The resulting consumption may lead to a more inefficient generation curve and thus deteriorate the system performance. This basic principle always holds, providing insights for extension to more general settings.

C. Mitigation Method: A Tariff Scheme

In this subsection, we focus on how to mitigate the negative impact of an aggregator's strategic behavior. The corollary below serves as the starting point of our methodology design.

Corollary 1: If there is no newly introduced congestion, the solution can be guaranteed to be at least Triple-win.

Proof: According to the proof of Theorem 1, it always holds that $\mu_{tl}^{+*} \le \mu_{tl}^{+0}$ under the Quadruple-win and Triple-win scenarios. Meanwhile it always holds that $\mu_{tl}^{+*} \ge \mu_{tl}^{+0}$ under the Double-win and Single-win scenarios. Therefore, if there is no newly introduced congestion, i.e., $\mu_{tl}^{+*} \le \mu_{tl}^{+0}$, the solution can be guaranteed to be either Triple-win or Quadruple-win. ■

Note that in general settings with multiple generators and aggregators, we will not have explicit conditions for the worst case where the generation cost increases. However, inspired by Corollary 1, it is reasonable to avoid the increase of the generation cost by preventing the submitted bids from introducing additional congestion. Specifically, the ISO can charge a congestion penalty for the newly introduced congestion from the strategic DSF aggregators. Taking the congestion penalty into consideration, the cost minimization problem for DSF aggregator $i \in \mathcal{N}$ becomes

$$\min_{\boldsymbol{x}_{\cdot i},\boldsymbol{l}_{\cdot i},\boldsymbol{u}_{\cdot i},\boldsymbol{x}^-_{\cdot i},\boldsymbol{x}^+_{\cdot i}} \quad \mathcal{C}_i^U\big(\boldsymbol{l}_{\cdot i},\boldsymbol{u}_{\cdot i},\boldsymbol{x}^-_{\cdot i},\boldsymbol{x}^+_{\cdot i}\big) + \boldsymbol{\pi}^T_{\cdot i}\boldsymbol{x}_{\cdot i} + b_i\big\|\big(\boldsymbol{\mu}^+ + \boldsymbol{\mu}^- - \boldsymbol{\mu}^{+0} - \boldsymbol{\mu}^{-0}\big)^+\boldsymbol{c}\big\|_1$$
$$s.t. \quad \text{Constraints (2d)–(2g), (3b)–(3c).}$$

The congestion charge is the excess payments received from the loads over the dispatch payments to the generators, and it results entirely from congestion [28]. Thus, we can use the congestion charge to characterize the congestion level of transmission networks with an economic rationale. This can be mathematically represented as $\|(\boldsymbol{\mu}^+ + \boldsymbol{\mu}^-)\boldsymbol{c}\|$. Compared to the case without a congestion penalty, the third term in the above objective is added to characterize the additional congestion. The penalty coefficient b_i is a constant predetermined by the ISO, and $\{\boldsymbol{\mu}^{+0}, \boldsymbol{\mu}^{-0}\}$ are the optimal solutions of the benchmark case. For the multiple aggregators case, the benchmark used for each aggregator can be different by only treating that particular aggregator as non-strategic. In practice, these benchmarks are approximated by the system operator based on historical data. We will not discuss the detailed approximation in this article.

Moreover, to balance the system budget, the additional charge corresponding to the congestion penalty collected from the strategic DSF aggregator will be used to compensate the fixed loads, whose payments will otherwise increase due to the heavier congestion. Specifically, the fixed load at bus n at time t is allowed to reduce its payment by $\frac{d_{tn}(H_{lg}-H_{ln})}{\sum_{n'\in\mathcal{N}} d_{tn'}(H_{lg}-H_{ln'})}\sum_{l\in\mathcal{L}_c} b_i(\mu_{tl}^{+*} - \mu_{tl}^{+0})c_l$.

Proposition 1: With the congestion penalty, the increase of the generation cost and the payment of the fixed load in the Single-win scenario can be mitigated.

Proof: Under the congestion penalty scheme, the change in the strategic aggregator's payment, compared with the benchmark case, can be given as

$$\begin{aligned}\Delta P_i^{CP} &= \sum\nolimits_{t\in\mathcal{T}_c}\Big[2A\big(g_t^* - \overline{g}^*\big)\Delta x_{ti} \\ &\quad + \sum_{l\in\mathcal{L}_c}\Big(\mu_{tl}^{+*}x_{ti}^* - \mu_{tl}^{+0}x_{ti}^0\Big)\big(H_{lg} - H_{li}\big) \\ &\quad + \sum\nolimits_{l\in\mathcal{L}_c} b_i\Big(\mu_{tl}^{+*} - \mu_{tl}^{+0}\Big)c_l\Big]. \end{aligned} \tag{8}$$

After adding the congestion penalty charge, i.e., the third term in (8), the strategic aggregator will face a heavier payment by introducing additional congestion. The aggregator has to trade off between a lower payment and a higher congestion penalty, and thus it will select the bids that will not introduce as much congestion as the case without a congestion penalty. Consequently, the probability that the generation cost will increase will become lower.

The change in the payment of the fixed load at bus n, compared with the benchmark case, can be written as

$$\begin{aligned}\Delta P_n^{CP} &= \sum\nolimits_{t\in\mathcal{T}_c}\Bigg[2A\big(d_{tn} - \overline{d}_n\big)\Delta g_t \\ &\quad + \sum\nolimits_{l\in\mathcal{L}_c}\Big(\mu_{tl}^{+*} - \mu_{tl}^{+0}\Big)d_{tn}\big(H_{lg} - H_{ln}\big) \\ &\quad - \frac{d_{tn}\big(H_{lg}-H_{ln}\big)}{\sum_{n'\in\mathcal{N}} d_{tn'}\big(H_{lg}-H_{ln'}\big)}\sum\nolimits_{l\in\mathcal{L}_c} b_i\Big(\mu_{tl}^{+*}-\mu_{tl}^{+0}\Big)c_l\Bigg]. \end{aligned} \tag{9}$$

The second term in (9) becomes lower than that of the case without a congestion penalty due to the mitigation of the newly introduced congestion. Additionally, the third term, i.e., the compensation term, helps mitigate the increase of the fixed load payment as well. ■

Proposition 2: The increase in payment of the fixed load at bus n because of the newly introduced congestion will be fully compensated for by the additional charge from the strategic DSF aggregator if $b_i \ge \frac{\sum_{n'\in\mathcal{N}} d_{tn'}(H_{lg}-H_{ln'})}{c_l}, \forall t \in \mathcal{T}_c$.

Proof: The first term in (9) is the change in payment due to the variation of the generation profile. Its increase is not the direct result of the change in generation cost and is not necessary to be compensated for by the strategic aggregator. The second term is the change in payment due to the variation of congestion. Its increase is the direct result of the newly introduced congestion and is necessary to be compensated for by the strategic aggregator. To let the compensation fully cover the increased payment due to the heavier congestion, we can achieve the conditions for the parameter b_i, as stated in Proposition 2, by setting the third term to be no less than the second term in (9). ■

IV. Case Study

Following the results developed in the previous sections, we now proceed to characterize the potential impact of strategic bidding via case studies. The time duration of each time slot is set to be one hour, and T = 24. We provide an illustrative example based on a 6-bus network modified from [29].

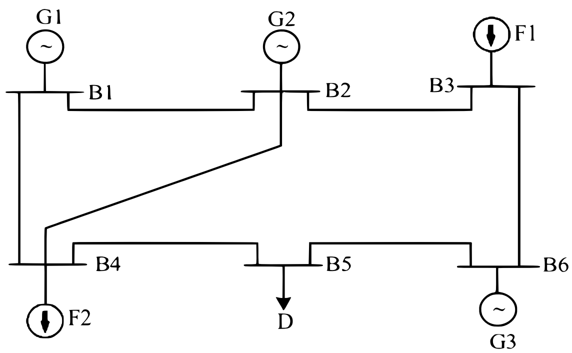


Fig. 1. Topology of the 6-bus test system.

TABLE I
NETWORK PARAMETERS

From Bus	To Bus	$X(pu)$	Flow Limit(MW)
1	2	0.170	41.7
1	4	0.258	39.6
2	3	0.037	39.6
2	4	0.197	42.7
3	6	0.018	37.5
4	5	0.037	37.5
5	6	0.140	37.5

TABLE II
SHIFT FACTOR MATRIX H

Bus	1	2	3	4	5	6
line 1-2	0	-0.68	-0.65	-0.48	-0.51	-0.63
line 1-4	0	-0.32	-0.35	-0.52	-0.49	-0.37
line 2-3	0	0.15	-0.75	-0.22	-0.32	-0.70
line 2-4	0	0.17	0.1	-0.26	-0.19	0.07
line 3-6	0	0.15	0.25	-0.22	-0.32	-0.70
line 4-5	0	-0.15	-0.25	0.22	-0.68	-0.30
line 5-6	0	-0.15	-0.25	0.22	0.32	-0.30

TABLE III
GENERATOR PARAMETERS

	$A(\$/MW^2h)$	$B(\$/MWh)$	Capacity(MW)
G_1	0.03	7	110
G_2	0.07	10	50
G_3	0.05	8	12.5

The system topology is shown in Fig. 1. The network parameters are given in Table I, and the calculated shift factors are shown in Table II. There are three generators G1, G2, and G3, located at bus #1, #2, and #6, respectively. The generator parameters are given in Table III. In our simulation, the quadratic production costs have been linearized through a piecewise linear approximation with three segments. The only base load D at bus #5 is assumed to follow the residential load profile from OpenEI [30], shown as the black solid line in Fig. 2(b). The two DSF aggregators F1 and F2 are respectively located at buses #3 and #4. They are assumed to share the same preferred flexibility bounds for cumulative energy and instantaneous power consumption. The upper and lower bounds are respectively shown as the green dashed and blue dashed-dotted lines in Fig. 2. To analyze the performance of strategic bidding, the comparison between different locations of DSF aggregators, and the impacts of introducing congestion penalty charges, we consider the following five cases:

Case 1: The benchmark case (both F1 and F2 non-strategic).

Case 2: F1 is a strategic aggregator, while F2 is non-strategic.

Case 3: F2 is a strategic aggregator, while F1 is non-strategic.

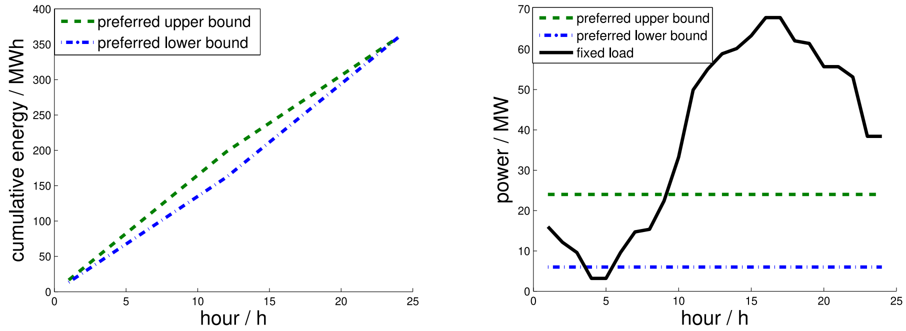
Case 4: F1 is a strategic aggregator, while F2 is non-strategic under the congestion penalty scheme.

Case 5: F2 is a strategic aggregator, while F1 is non-strategic under the congestion penalty scheme.

A. The Performance of Strategic Bidding

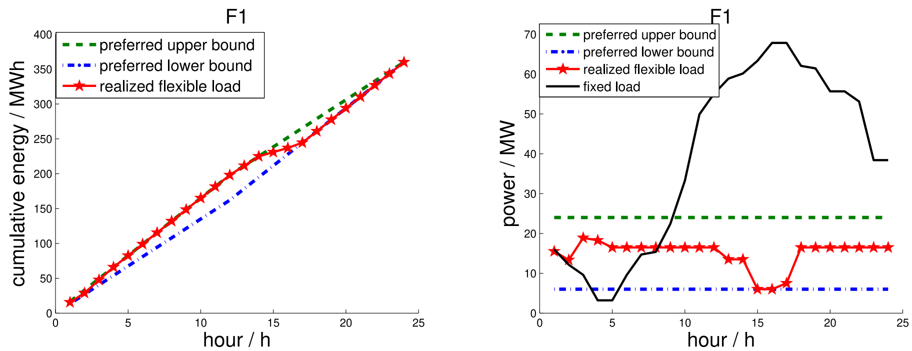
We first simulate different strategic bidding cases without a congestion penalty (Cases 2 and 3). Fig. 3 shows the flexibility bounds bidding and dispatch of the DSF aggregators for Case 3 as an illustrative example. F1 is non-strategic and adopts the exact preferences as the bids. As shown in Fig. 3 (a) and (b), the upper and lower bounds of cumulative energy and instantaneous power consumption strictly follow the preferred ones in Fig. 2. The dispatch results (realized flexible loads) shown as the red solid lines with stars are restricted within the flexible region. F2 is strategic and chooses the optimal bids (the green solid lines with circles and blue solid lines with diamonds) within the physical limits (the green dashed lines with crosses and blue dashed-dotted lines with crosses). As shown in Fig. 3 (c) and (d), the optimal bids are different from the preferred bounds. For the cumulative energy consumption in Fig. 3 (c), a bar plot is shown at the bottom to make the results more clear. Specifically, the green (blue) bars represent the difference between the optimal upper (lower) bound and the preferred upper (lower) bound. It can be seen that the optimal upper bound is higher than the preferred one at hours 4–19, and the optimal lower bound is lower than the preference at hours 21–23. For the instantaneous power consumption, the optimal upper bound is higher, while the optimal lower bound is lower than the preferences. Note that the optimal flexible region is not always broader than the preferred one. For example, in Fig. 3 (c), the optimal upper bound is even lower than the preference at hour 21 and the optimal lower bound is even higher at hours 12, 14, and 15. The dispatch results are forced to the corresponding bounds at these hours. In this sense, the strategic DSF aggregator manipulates the consumption curve through strategic bidding of the flexible region instead of simply increasing the flexibility.

We next compare the system performances with different deviation coefficients a_i in the strategic bidding cases. Note that a_1 and a_2 are the coefficient of deviation from the preferred flexibility bounds in the utility costs of F1 and F2, respectively. In Case 3, F1, the non-strategic aggregator, has to follow the preferred bounds strictly, which means a_1 is set to infinity. The decrease of a_2 means that larger deviation is allowed with an acceptable utility cost. As shown in the bar plots of Fig. 3 (c) and Fig. 4, the optimal bids of the strategic aggregator F2 follow the preferred bounds more as a_2 increases. In the extreme case, the optimal bids are forced to strictly follow the preferred bounds when a_2 is infinitely large, which degenerates to Case 1 as a result. The system performances of Cases 1, 2, and 3 with different deviation coefficients are shown in Table IV in terms of

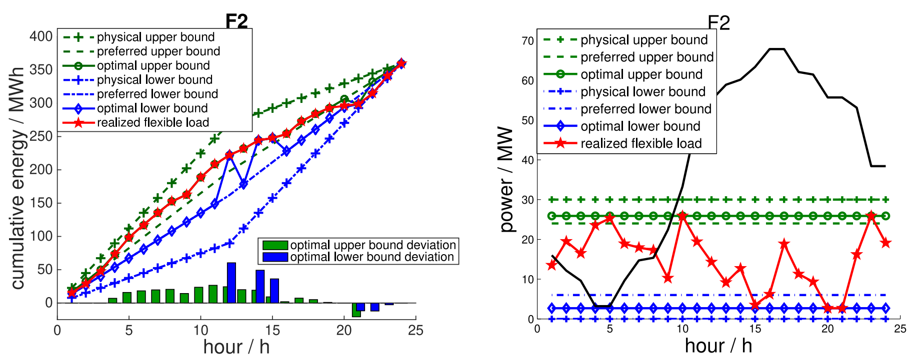


(a) Cumulative energy. (b) Instantaneous power.

Fig. 2. The base load and preferred flexibility bounds of DSF aggregators.

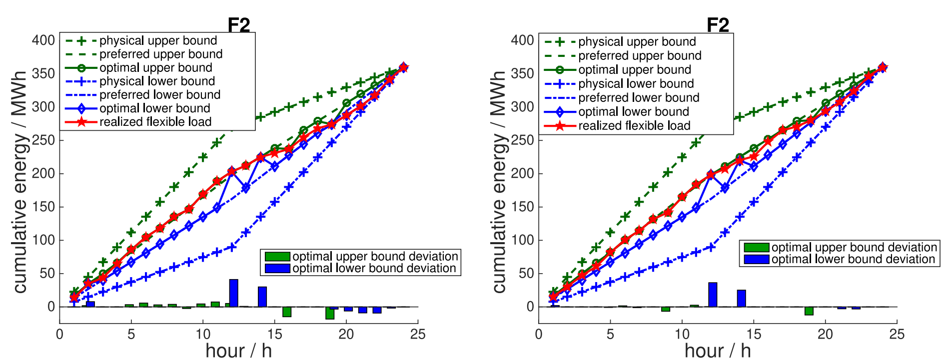


(a) Cumulative energy. (b) Instantaneous power.



(c) Cumulative energy. (d) Instantaneous power.

Fig. 3. The bidding and dispatch of F1 and F2 in Case 3 with $a_2 = 0.01$.



(a) Cumulative energy ($a_2 = 0.04$). (b) Cumulative energy ($a_2 = 0.1$).

Fig. 4. The bidding and dispatch of F2 in Case 3 with different a_2.

the generation costs, the payments of different load aggregators, and the congestion charges. It can be observed that the payment of F2 goes down with the decrease of a_2 in Case 3. Similarly, the payment of F1 goes down with the decrease of a_1 in Case 2. This is because the strategic DSF aggregator cares more about the energy purchasing cost than

TABLE IV
SYSTEM PERFORMANCES WITH DIFFERENT DEVIATION COEFFICIENTS

	a_1	a_2	Gen. Cost	Payments			Cong. Charge
				F1	F2	D	
Case 1	∞	∞	14627	3574	3501	10238	532
Case 2	0.1	∞	14611	3468	3444	9667	177
	0.04	∞	14596	3453	3443	9665	177
	0.01	∞	14596	3453	3443	9665	177
Case 3	∞	0.1	14667	3768	3133	14647	4555
	∞	0.04	14719	3806	3130	15928	5178
	∞	0.01	14692	3885	3123	16390	6165

TABLE V
CONDITIONS CHECKING FOR SINGLE-WIN SCENARIO IN CASE 3

time $t(h)$	13	14	15	16	17	18	19	20	21
$-\frac{c_l-f_{tl}^0}{H_{li}-H_{lj}}$	0	0	0	0	0	0	0	-1.6	-4.2
Δx_{ti}	-2.3	0	-12.2	-15.5	-2.9	-6.6	-4.4	-3.3	-9.3

the utility cost when the value of the deviation coefficient is small.

B. The Impact of the Locations of the DSF Aggregators

The only difference between Case 2 and Case 3 is that the locations of the strategic DSF aggregator and the non-strategic one are shifted. As observed from the simulation results, the only possible congested line is line 4-5. According to Table II, for this particular line, the shift factor of the fixed load at bus #5 is -0.68, which is the smallest among all buses. For the three generators at buses #1 (the reference bus), #2, and #6, the shift factors are 0, -0.15, and -0.3, respectively. The shift factor of F1 at bus #3 is -0.25, larger than that of one of the generators but smaller than those of the other two, while that of F2 at bus #4 is 0.22, larger than those of all the generators. Although the test model does not fit all the assumptions in Section III, we get insights into the practical applications of the theoretical conclusions on general system models.

We compare the system performances of Cases 1, 2, and 3 in Table IV. All the winners are highlighted in yellow. In Case 2, whichever a_1 value is chosen, the system generation cost and the payments of the non-strategic aggregator F2 and the fixed load D are lower than those under Case 1. The congestion is also mitigated by comparing the congestion charge, which is consistent with Corollary 1. The strategic bidding of F1 reaches a Quadruple-win situation. Generalized from Scenario-II in Theorem 1, this can be explained as follows: The shift factor of the strategic DSF aggregator is lower than those of some generators, while the shift factor of the non-strategic aggregator is higher than those of all the generators.

In contrast, in Case 3, whichever a_2 value is chosen, the generation cost and the payments of the non-strategic aggregator F1 and fixed load D are even higher than those under Case 1. The congestion also becomes even heavier. The strategic bidding of F2 results in a Single-win situation, with F2 being the only winner. Generalized from Scenario-IV in Theorem 1, this is because the shift factor of the strategic DSF aggregator is higher than those of all the generators, while the shift factor of the non-strategic aggregator is higher than that of some generator. Moreover, we calculate $-\frac{c_l-f_{tl}^0}{H_{li}-H_{lj}}$ (l is the only possible congested line 4-5) and Δx_{ti} ($t \in \mathcal{T}_c$) in Table V, and check the conditions for the Single-win scenario in Theorem 1. It can be seen that at each congested time slot, $\Delta x_{ti} < -\frac{c_l-f_{tl}^0}{H_{li}-H_{lj}}$ is always satisfied. Thus, the conditions $H_{li}(= 0.22) > H_{lj}(= -0.25)$ and $\sum_{t\in\mathcal{T}_c} \Delta x_{ti} < -\frac{\sum_{t\in\mathcal{T}_c}(c_l-f_{tl}^0)}{H_{li}-H_{lj}}$, $l \in \mathcal{L}_c$ are guaranteed.

TABLE VI
SYSTEM PERFORMANCES UNDER CONGESTION PENALTY SCHEME

	b	Gen. Cost	Payments			Cong. Charge
			F1	F2	D	
Case 1	-	14627	3574	3501	10238	532
Case 2	0	14596	3453	3443	9665	177
Case 4	0.1	14596	3453	3443	9665	177
	1	14596	3580	3570	9949	0
Case 3	0	14719	3806	3130	15928	5178
Case 5	0.1	14625	3540	3317	10921	1340
	1	14625	3501	3544	9710	0

C. The Performance of the Congestion Penalty Scheme

We further simulate different strategic bidding cases under the congestion penalty scheme (Cases 4 and 5). The system performances with different penalty coefficients b (suppose $b = b_1 = b_2$) are shown in Table VI, where the winners are highlighted. The decrease of b means a smaller penalty for newly introduced congestion. In the extreme case when b becomes 0, Case 4 and Case 5 respectively deteriorate to Case 2 and Case 3. Table VI also shows Case 1, Case 2 ($a_1 = 0.04$), and Case 3 ($a_2 = 0.04$) for comparison. By comparing Case 4 with Case 2, a small $b = 0.1$ results in the same system performance. With a large $b = 1$, the congestion is mitigated and disappears at the expense of slightly increasing the payments of F1, F2, and D. By comparing Case 5 with Case 3, a small $b = 0.1$ helps decrease the congestion charge from 5178 to 1340. Although it is still higher than the congestion charge in Case 1, the system generation cost and the payment of F1 drop to lower values than those under the benchmark case. With a large $b = 1$, the congestion disappears. All the system performance metrics, including the generation cost, and the payments of F1 and D, become lower than those under Case 1, except that the payment of F2 becomes higher. It can be seen that $b = 0.1$ is a proper choice in Case 5 to trade off between avoiding the negative effects of the strategic bidding and encouraging the self-profit of the strategic DSF aggregator.

V. CONCLUDING REMARKS

Understanding the potential for market manipulation by flexible load aggregators is crucial for maintaining the efficiency of electricity markets. In this article, we characterized the cost saving an aggregator can make by strategically bidding the flexible region in the forward market as the outcome of a bi-level optimization problem. Through theoretical analysis and case studies, we demonstrated that the DSF aggregators can indeed degrade the system performance. These impacts are closely related to the bus locations of different market participants. Moreover, we showed that the increase of the generation cost and the fixed load's payment always occur when there exists newly introduced congestion. Thus, we proposed a congestion penalty scheme for the operator to avoid the potential adverse effects. Although the analytical results in this article are derived under some technical assumptions, we view this work as an initial step in anticipating the negative effects of flexible load aggregators beforehand, based only on the network information. One limitation is that we only consider one strategic and one non-strategic aggregator in the theoretical derivation. Extending the analysis to the case of multiple aggregators in the market is an interesting direction for future research.

Appendix A
Formulation and Solution Method of the MPEC

This appendix gives the detailed formulation and solution method of the MPEC derived from the single-level problem (3). After replacing the lower problem (1) with its equivalent KKT conditions in problem (3), we can get the MPEC as follows:

$$\min_{\mathcal{S}} \ \mathcal{C}_i^U\left(\boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}_{\cdot i}^-, \boldsymbol{x}_{\cdot i}^+\right) + \boldsymbol{\pi}_{\cdot i}^T \boldsymbol{x}_{\cdot i} \tag{10a}$$

s.t. Constraints (2b)-(2g),

$$\boldsymbol{\pi} = \boldsymbol{\lambda}\mathbf{1}^T + \left(\boldsymbol{\mu}^- - \boldsymbol{\mu}^+\right)\boldsymbol{H}, \tag{10b}$$

$$(\boldsymbol{g} - \boldsymbol{d} - \boldsymbol{x})\mathbf{1} = \mathbf{0}, \tag{10c}$$

$$\frac{\partial \boldsymbol{\mathcal{C}}^G(\boldsymbol{g})}{\partial \boldsymbol{g}} - \boldsymbol{\lambda}\mathbf{1}^T + \left(\boldsymbol{\mu}^+ - \boldsymbol{\mu}^-\right)\boldsymbol{H} + \boldsymbol{\gamma}^{g+} - \boldsymbol{\gamma}^{g-} = \mathbf{0}, \tag{10d}$$

$$\boldsymbol{\lambda}\mathbf{1}^T + \left(\boldsymbol{\mu}^- - \boldsymbol{\mu}^+\right)\boldsymbol{H} + \boldsymbol{L}^T\left(\boldsymbol{\gamma}^{e+} - \boldsymbol{\gamma}^{e-}\right) + \boldsymbol{\gamma}^{p+} - \boldsymbol{\gamma}^{p-} = \mathbf{0}, \tag{10e}$$

$$\mathbf{0} \le \boldsymbol{\mu}^- \perp \boldsymbol{H}(\boldsymbol{g} - \boldsymbol{d} - \boldsymbol{x})^T + \boldsymbol{c}\mathbf{1}^T \ge \mathbf{0}, \tag{10f}$$

$$\mathbf{0} \le \boldsymbol{\mu}^+ \perp \boldsymbol{c}\mathbf{1}^T - \boldsymbol{H}(\boldsymbol{g} - \boldsymbol{d} - \boldsymbol{x})^T \ge \mathbf{0}, \tag{10g}$$

$$\mathbf{0} \le \boldsymbol{\gamma}^{g-} \perp \boldsymbol{g} - \boldsymbol{g}^- \ge \mathbf{0}, \tag{10h}$$

$$\mathbf{0} \le \boldsymbol{\gamma}^{g+} \perp \boldsymbol{g}^+ - \boldsymbol{g} \ge \mathbf{0}, \tag{10i}$$

$$\mathbf{0} \le \boldsymbol{\gamma}^{e-} \perp \boldsymbol{L}\boldsymbol{x} - \boldsymbol{l} \ge \mathbf{0}, \tag{10j}$$

$$\mathbf{0} \le \boldsymbol{\gamma}^{e+} \perp \boldsymbol{u} - \boldsymbol{L}\boldsymbol{x} \ge \mathbf{0}, \tag{10k}$$

$$\mathbf{0} \le \boldsymbol{\gamma}^{p-} \perp \boldsymbol{x} - \boldsymbol{x}^- \ge \mathbf{0}, \tag{10l}$$

$$\mathbf{0} \le \boldsymbol{\gamma}^{p+} \perp \boldsymbol{x}^+ - \boldsymbol{x} \ge \mathbf{0}, \tag{10m}$$

where the decision set $\mathcal{S} = \{\boldsymbol{g}, \boldsymbol{x}, \boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}_{\cdot i}^-, \boldsymbol{x}_{\cdot i}^+, \boldsymbol{\lambda}, \boldsymbol{\mu}^-, \boldsymbol{\mu}^+, \boldsymbol{\gamma}^{g-}, \boldsymbol{\gamma}^{g+}, \boldsymbol{\gamma}^{e-}, \boldsymbol{\gamma}^{e+}, \boldsymbol{\gamma}^{p-}, \boldsymbol{\gamma}^{p+}\}$ includes the power dispatch $\{\boldsymbol{g}, \boldsymbol{x}\}$, flexibility bound bidding $\{\boldsymbol{l}_{\cdot i}, \boldsymbol{u}_{\cdot i}, \boldsymbol{x}_{\cdot i}^-, \boldsymbol{x}_{\cdot i}^+\}$, and dual variables set $\{\boldsymbol{\lambda}, \boldsymbol{\mu}^-, \boldsymbol{\mu}^+, \boldsymbol{\gamma}^{g-}, \boldsymbol{\gamma}^{g+}, \boldsymbol{\gamma}^{e-}, \boldsymbol{\gamma}^{e+}, \boldsymbol{\gamma}^{p-}, \boldsymbol{\gamma}^{p+}\}$. The notation $0 \le P \perp Q \ge 0$ for two scalars, P and Q, indicates $P \ge 0$, $Q \ge 0$, and $P \cdot Q = 0$. This holds for each element when it comes to matrices.

The single-level reformulation (10) of the aggregator's bi-level optimization is hard to solve due to the following two aspects of non-convexities: *i) the complementary conditions* (10f)–(10m)*, and ii) the bilinear term* $\boldsymbol{\pi}_{\cdot i}^T \boldsymbol{x}_{\cdot i}$ *in the objective function*. The detailed linearizations are shown below.

i) The first group of non-convex constraints can be linearized using the well-known Big-M method: $0 \le P \perp Q \ge 0 \Leftrightarrow 0 \le P \le z \cdot M, 0 \le Q \le (1-z) \cdot M, z \in \{0, 1\}$. Specifically, the complementary conditions (10f)–(10m) can be transformed into the following set of constraints:

$$\begin{aligned}
&\mathbf{0} \le \boldsymbol{\mu}^- \le \boldsymbol{z}^1 \cdot \boldsymbol{M}^1,\\
&\mathbf{0} \le \boldsymbol{H}(\boldsymbol{g} - \boldsymbol{d} - \boldsymbol{x})^T + \boldsymbol{c}\mathbf{1}^T \le \left(\boldsymbol{I} - \boldsymbol{z}^1\right) \cdot \boldsymbol{M}^1,\\
&\mathbf{0} \le \boldsymbol{\mu}^+ \le \boldsymbol{z}^2 \cdot \boldsymbol{M}^2,\\
&\mathbf{0} \le \boldsymbol{c}\mathbf{1}^T - \boldsymbol{H}(\boldsymbol{g} - \boldsymbol{d} - \boldsymbol{x})^T \le \left(\boldsymbol{I} - \boldsymbol{z}^2\right) \cdot \boldsymbol{M}^2,\\
&\mathbf{0} \le \boldsymbol{\gamma}^{g-} \le \boldsymbol{z}^3 \cdot \boldsymbol{M}^3,\\
&\mathbf{0} \le \boldsymbol{g} - \boldsymbol{g}^- \le \left(\boldsymbol{I} - \boldsymbol{z}^3\right) \cdot \boldsymbol{M}^3,\\
&\mathbf{0} \le \boldsymbol{\gamma}^{g+} \le \boldsymbol{z}^4 \cdot \boldsymbol{M}^4,\\
&\mathbf{0} \le \boldsymbol{g}^+ - \boldsymbol{g} \le \left(\boldsymbol{I} - \boldsymbol{z}^4\right) \cdot \boldsymbol{M}^4,\\
&\mathbf{0} \le \boldsymbol{\gamma}^{e-} \le \boldsymbol{z}^5 \cdot \boldsymbol{M}^5,\\
&\mathbf{0} \le \boldsymbol{L}\boldsymbol{x} - \boldsymbol{l} \le \left(\boldsymbol{I} - \boldsymbol{z}^5\right) \cdot \boldsymbol{M}^5,\\
&\mathbf{0} \le \boldsymbol{\gamma}^{e+} \le \boldsymbol{z}^6 \cdot \boldsymbol{M}^6,\\
&\mathbf{0} \le \boldsymbol{u} - \boldsymbol{L}\boldsymbol{x} \le \left(\boldsymbol{I} - \boldsymbol{z}^6\right) \cdot \boldsymbol{M}^6,\\
&\mathbf{0} \le \boldsymbol{\gamma}^{p-} \le \boldsymbol{z}^7 \cdot \boldsymbol{M}^7,\\
&\mathbf{0} \le \boldsymbol{x} - \boldsymbol{x}^- \le \left(\boldsymbol{I} - \boldsymbol{z}^7\right) \cdot \boldsymbol{M}^7,\\
&\mathbf{0} \le \boldsymbol{\gamma}^{p+} \le \boldsymbol{z}^8 \cdot \boldsymbol{M}^8,\\
&\mathbf{0} \le \boldsymbol{x}^+ - \boldsymbol{x} \le \left(\boldsymbol{I} - \boldsymbol{z}^8\right) \cdot \boldsymbol{M}^8,\\
&\quad \boldsymbol{z}^1, \boldsymbol{z}^2, \boldsymbol{z}^3, \boldsymbol{z}^4, \boldsymbol{z}^5, \boldsymbol{z}^6, \boldsymbol{z}^7, \boldsymbol{z}^8 \in \{\mathbf{0}, \mathbf{1}\}.
\end{aligned}$$

ii) To tackle the non-convexity of the bilinear term $\boldsymbol{\pi}_{\cdot i}^T \boldsymbol{x}_{\cdot i}$ in the objective function, we can leverage the KKT conditions and obtain an equivalent linear expression:

$$\begin{aligned}
\boldsymbol{\pi}_{\cdot i}^T \boldsymbol{x}_{\cdot i} = &\sum_{t,n}\left(\frac{\partial \mathcal{C}^G}{\partial g_{tn}} g_{tn} - \gamma_{tn}^{g-} g_{tn}^- + \gamma_{tn}^{g+} g_{tn}^+\right)\\
&- \sum_{t,n}\left(\lambda_t d_{tn} - \sum_l\left(\mu_{tl}^+ - \mu_{tl}^-\right)H_{ln} d_{tn}\right)\\
&- \sum_{t,n \ne i}\left(\gamma_{tn}^{e-} l_{tn} - \gamma_{tn}^{e+} u_{tn} + \gamma_{tn}^{p-} x_{tn}^- - \gamma_{tn}^{p+} x_{tn}^+\right)\\
&+ \sum_{t,l}\left(\mu_{tl}^+ + \mu_{tl}^-\right)c_l.
\end{aligned}$$

The detailed derivation is given as follows:

$$\begin{aligned}
&\boldsymbol{\pi}_{\cdot i}^T \boldsymbol{x}_{\cdot i}\\
&= \sum_t\left(\lambda_t - \sum_l\left(\mu_{tl}^+ - \mu_{tl}^-\right)H_{li}\right)\left(\sum_n g_{tn} - \sum_n d_{tn} - \sum_{n \ne i} x_{tn}\right)\\
&= \sum_{t,n}\left(\lambda_t g_{tn} - \sum_l\left(\mu_{tl}^+ - \mu_{tl}^-\right)H_{li} g_{tn}\right)\\
&\quad - \sum_{t,n}\left(\lambda_t d_{tn} - \sum_l\left(\mu_{tl}^+ - \mu_{tl}^-\right)H_{li} d_{tn}\right)\\
&\quad - \sum_{t,n \ne i}\left(\lambda_t x_{tn} - \sum_l\left(\mu_{tl}^+ - \mu_{tl}^-\right)H_{li} x_{tn}\right)
\end{aligned}$$

$$
\begin{aligned}
&= \sum_{t,n}\Bigg(\frac{\partial \mathcal{C}^G}{\partial g_{tn}} g_{tn} + \sum_l \big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{ln} g_{tn} - \gamma_{tn}^{g-} g_{tn} \\
&\qquad + \gamma_{tn}^{g+} g_{tn} - \sum_l \big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{li} g_{tn}\Bigg) \\
&\quad - \sum_{t,n}\Bigg(\lambda_t d_{tn} - \sum_l \big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{li} d_{tn}\Bigg) \\
&\quad - \sum_{t,n\neq i}\Bigg(\sum_l \big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{ln} x_{tn} + \big(\boldsymbol{L}\boldsymbol{\gamma}_n^{e-}\big)_t x_{tn} - \big(\boldsymbol{L}\boldsymbol{\gamma}_n^{e+}\big)_t x_{tn} \\
&\qquad + \gamma_{tn}^{p-} x_{tn} - \gamma_{tn}^{p+} x_{tn} - \sum_l \big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{li} x_{tn}\Bigg) \\
&= \sum_{t,n}\Bigg(\frac{\partial \mathcal{C}^G}{\partial g_{tn}} g_{tn} - \gamma_{tn}^{g-} g_{tn} + \gamma_{tn}^{g+} g_{tn}\Bigg) - \sum_{t,n}(\lambda_t d_{tn}) \\
&\quad - \sum_{t,n\neq i}\Big(\big(\boldsymbol{L}\boldsymbol{\gamma}_n^{e-}\big)_t x_{tn} - \big(\boldsymbol{L}\boldsymbol{\gamma}_n^{e+}\big)_t x_{tn} + \gamma_{tn}^{p-} x_{tn} - \gamma_{tn}^{p+} x_{tn}\Big) \\
&\quad + \sum_{t,n,l}\big(\mu_{tl}^+ - \mu_{tl}^-\big)(H_{ln} g_{tn} - H_{li} g_{tn}) + \sum_{t,n,l}\big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{li} d_{tn} \\
&\quad - \sum_{t,n\neq i,l}\big(\mu_{tl}^+ - \mu_{tl}^-\big)(H_{ln} x_{tn} - H_{li} x_{tn}) \\
&= \sum_{t,n}\Bigg(\frac{\partial \mathcal{C}^G}{\partial g_{tn}} g_{tn} - \gamma_{tn}^{g-} g_{tn} + \gamma_{tn}^{g+} g_{tn}\Bigg) - \sum_{t,n}(\lambda_t d_{tn}) \\
&\quad - \sum_{t,n\neq i}\Big(\big(\boldsymbol{L}\boldsymbol{\gamma}_n^{e-}\big)_t x_{tn} - \big(\boldsymbol{L}\boldsymbol{\gamma}_n^{e+}\big)_t x_{tn} + \gamma_{tn}^{p-} x_{tn} - \gamma_{tn}^{p+} x_{tn}\Big) \\
&\quad + \sum_{t,l}\big(\mu_{tl}^+ - \mu_{tl}^-\big)\Bigg(\sum_n H_{ln} g_{tn} - \sum_{n\neq i} H_{ln} x_{tn}\Bigg) \\
&\quad + \sum_{t,l}\big(\mu_{tl}^+ - \mu_{tl}^-\big)\Bigg(\sum_n H_{li} g_{tn} - \sum_n H_{li} d_{tn} - \sum_{n\neq i} H_{li} x_{tn}\Bigg) \\
&= \sum_{t,n}\Bigg(\frac{\partial \mathcal{C}^G}{\partial g_{tn}} g_{tn} - \gamma_{tn}^{g-} g_{tn}^- + \gamma_{tn}^{g+} g_{tn}^+\Bigg) - \sum_{t,n}(\lambda_t d_{tn}) \\
&\quad - \sum_{t,n\neq i}\Big(\gamma_{tn}^{e-} l_{tn} - \gamma_{tn}^{e+} u_{tn} + \gamma_{tn}^{p-} x_{tn}^- - \gamma_{tn}^{p+} x_{tn}^+\Big) \\
&\quad + \sum_{t,l}\Bigg(\mu_{tl}^+ c_l + \mu_{tl}^+ \sum_n H_{ln} d_{tn} + \mu_{tl}^+ H_{li} x_{ti} + \mu_{tl}^- c_l \\
&\qquad - \mu_{tl}^- \sum_n H_{ln} d_{tn} - \mu_{tl}^- H_{li} x_{ti}\Bigg) \\
&\quad - \sum_{t,l}\big(\mu_{tl}^+ - \mu_{tl}^-\big) H_{li} x_{ti} \\
&= \sum_{t,n}\Bigg(\frac{\partial \mathcal{C}^G}{\partial g_{tn}} g_{tn} - \gamma_{tn}^{g-} g_{tn}^- + \gamma_{tn}^{g+} g_{tn}^+\Bigg) - \sum_{t,n}(\lambda_t d_{tn}) \\
&\quad - \sum_{t,n\neq i}\Big(\gamma_{tn}^{e-} l_{tn} - \gamma_{tn}^{e+} u_{tn} + \gamma_{tn}^{p-} x_{tn}^- - \gamma_{tn}^{p+} x_{tn}^+\Big) \\
&\quad + \sum_{t,l}\Bigg(\big(\mu_{tl}^+ + \mu_{tl}^-\big) c_l + \big(\mu_{tl}^+ - \mu_{tl}^-\big) \sum_n H_{ln} d_{tn}\Bigg).
\end{aligned}
$$

If we assume the generation cost function to be quadratic or linear, the aggregator's strategic bidding problem (10) becomes a mixed-integer quadratic or linear program, respectively, after the above transformations. Thus, commercial solvers, such as Gurobi and CPLEX, can be applied to provide a global solution with acceptable computational time.

Appendix B
Derivation of the Changes

We derive the change of the strategic aggregator's payment, (5), as follows.

$$
\begin{aligned}
\Delta P_i &= \sum_t \lambda_t^* x_{ti}^* - \sum_t \lambda_t^0 x_{ti}^0 \\
&= \sum_t \Bigg(\frac{\partial \mathcal{C}^G\big(g_t^*\big)}{\partial g_t^*} + \sum_l \mu_{tl}^*\big(H_{lg} - H_{li}\big)\Bigg) x_{ti}^* \\
&\quad - \sum_t \Bigg(\frac{\partial \mathcal{C}^G\big(g_t^0\big)}{\partial g_t^0} + \sum_l \mu_{tl}^0\big(H_{lg} - H_{li}\big)\Bigg) x_{ti}^0 \\
&= 2A \sum_t \Big(g_t^* x_{ti}^* - g_t^0 x_{ti}^0\Big) \\
&\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big) \\
&= 2A \sum_t \Big(g_t^*\Big(x_{ti}^0 + \Delta x_{ti}\Big) - g_t^0 x_{ti}^0\Big) \\
&\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big) \\
&= 2A \sum_t \Big(\Delta g_t x_{ti}^0 + g_t^* \Delta x_{ti}\Big) \\
&\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big).
\end{aligned}
$$

Since the fixed loads are dominant according to assumption 3), we have $g_t^* \gg x_{ti}^0$. Meanwhile, Δg_t has the same order of magnitude as Δx_{ti}. Thus we have

$$
\begin{aligned}
\Delta P_i &= 2A \sum_t g_t^* \Delta x_{ti} + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big) \\
&= 2A \sum_{t\in\mathcal{T}_{uc}} g_t^* \Delta x_{ti} + 2A \sum_{t\in\mathcal{T}_c} g_t^* \Delta x_{ti} \\
&\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big).
\end{aligned}
$$

According to assumption 4), we can replace g_t^* with $\overline{g}$ in the first term. With $\sum_{t\in\mathcal{T}_{uc}} \overline{g}\Delta x_{ti} = -\sum_{t\in\mathcal{T}_c} \overline{g}\Delta x_{ti}$, it holds that

$$
\begin{aligned}
\Delta P_i &= -2A \sum_{t\in\mathcal{T}_c} \overline{g}\Delta x_{ti} + 2A \sum_{t\in\mathcal{T}_c} g_t^* \Delta x_{ti} \\
&\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big) \\
&= 2A \sum_{t\in\mathcal{T}_c} \big(g_t^* - \overline{g}\big)\Delta x_{ti} \\
&\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c} \Big(\mu_{tl}^* x_{ti}^* - \mu_{tl}^0 x_{ti}^0\Big)\big(H_{lg} - H_{li}\big).
\end{aligned}
$$

Then the change of the fixed load's payment at bus n, (6), can be derived as follows.

$$\begin{aligned}\Delta P_n &= \sum_t \lambda_t^* d_{tn} - \sum_t \lambda_t^0 d_{tn}\\ &= 2A\sum_t\left(g_t^* - g_t^0\right)d_{tn} + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c}\left(\mu_{tl}^* - \mu_{tl}^0\right)d_{tn}\left(H_{lg} - H_{ln}\right)\\ &= 2A\sum_{t\in\mathcal{T}_{uc}}\Delta g_t d_{tn} + 2A\sum_{t\in\mathcal{T}_c}\Delta g_t d_{tn}\\ &\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c}\left(\mu_{tl}^* - \mu_{tl}^0\right)d_{tn}\left(H_{lg} - H_{ln}\right).\end{aligned}$$

According to assumption 4), we can replace d_{tn} with $\overline{d}_n$ in the first term. With $\sum_{t\in\mathcal{T}_{uc}}\Delta g_t\overline{d}_n = -\sum_{t\in\mathcal{T}_c}\Delta g_t\overline{d}_n$, it holds that

$$\begin{aligned}\Delta P_n &= -2A\sum_{t\in\mathcal{T}_c}\Delta g_t\overline{d}_n + 2A\sum_{t\in\mathcal{T}_c}\Delta g_t d_{tn}\\ &\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c}\left(\mu_{tl}^* - \mu_{tl}^0\right)d_{tn}\left(H_{lg} - H_{ln}\right)\\ &= 2A\sum_{t\in\mathcal{T}_c}\left(d_{tn} - \overline{d}_n\right)\Delta g_t\\ &\quad + \sum_{t\in\mathcal{T}_c, l\in\mathcal{L}_c}\left(\mu_{tl}^* - \mu_{tl}^0\right)d_{tn}\left(H_{lg} - H_{ln}\right).\end{aligned}$$

Finally, the change of the power flow at line l at time t, (7), is

$$\begin{aligned}\Delta Flow_{tl} &= H_{lg}\Delta g_t - H_{li}\Delta x_{ti} - H_{lj}\Delta x_{tj}\\ &= H_{lg}\left(\Delta x_{ti} + \Delta x_{tj}\right) - H_{li}\Delta x_{ti} - H_{lj}\Delta x_{tj}\\ &= \left(H_{lg} - H_{li}\right)\Delta x_{ti} + \left(H_{lg} - H_{lj}\right)\Delta x_{tj}.\end{aligned}$$

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