Online Selection with Convex Costs

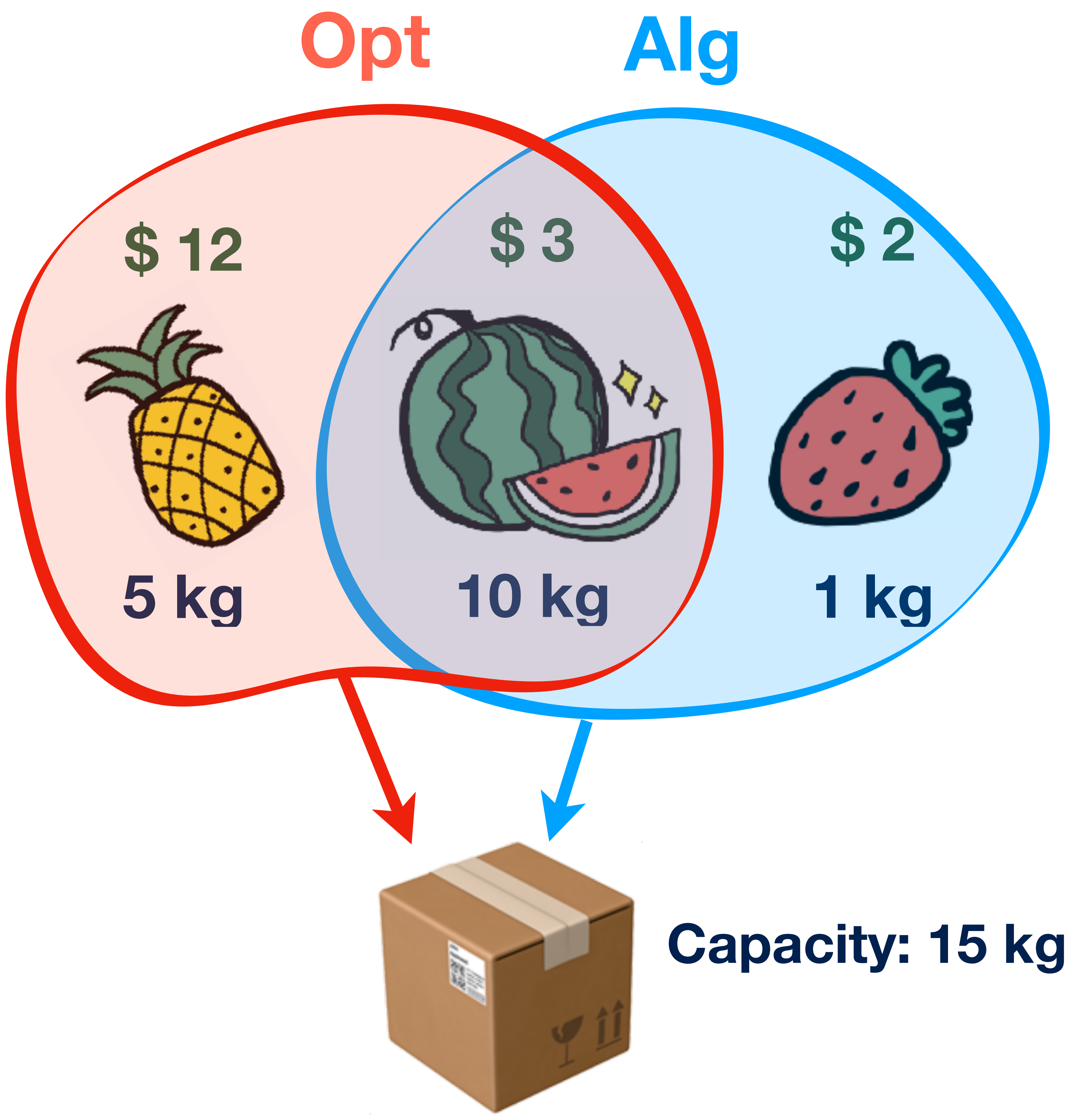
Xiaoqi Tan[1], Siyuan Yu[1], Raouf Boutaba[2]

[1]University of Alberta, Canada

[2]University of Waterloo, Canada

MAMA Workshop June 6, 2022

Example 1: Online Knapsack



$$\frac{\text{Profit (Opt)}}{\text{Profit (Alg)}} = \frac{3 + 12}{2 + 3} = 3$$

Achieve 1/3 of the best-possible profit (in hindsight)

Competitive Ratio (CR)

- A **deterministic online algorithm** is α-competitive if

$$\mathsf{Alg}(\sigma) \geq \frac{1}{\alpha} \cdot \mathsf{Opt}(\sigma), \quad \forall \sigma \iff \alpha = \max_{\sigma} \frac{\mathsf{Opt}(\sigma)}{\mathsf{Alg}(\sigma)}$$

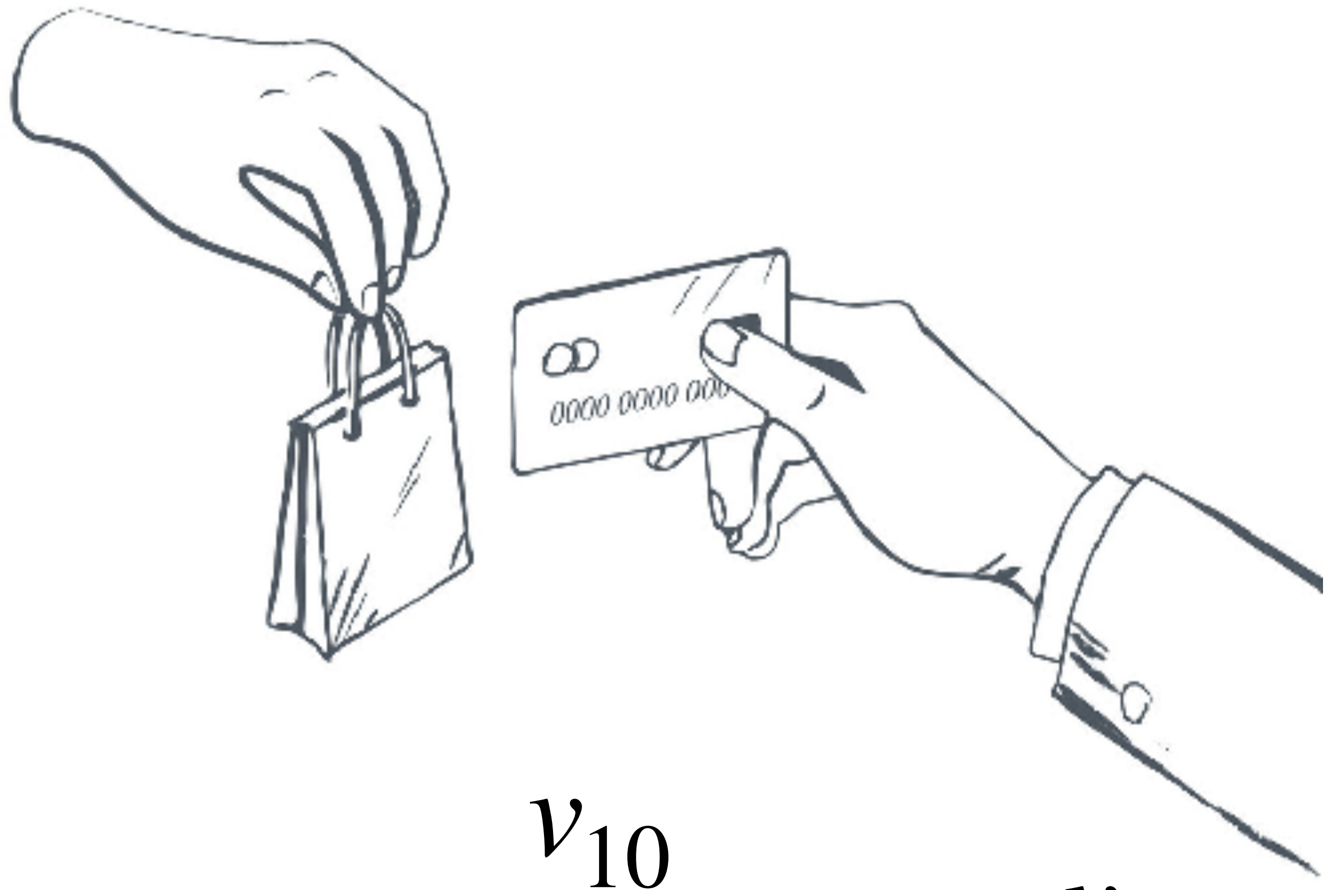
- A **randomized online algorithm** is α-competitive if

$$\mathbb{E}[\mathsf{Alg}(\sigma)] \geq \frac{1}{\alpha} \cdot \mathsf{Opt}(\sigma), \quad \forall \sigma \iff \alpha = \max_{\sigma} \frac{\mathsf{Opt}(\sigma)}{\mathbb{E}[\mathsf{Alg}(\sigma)]}$$

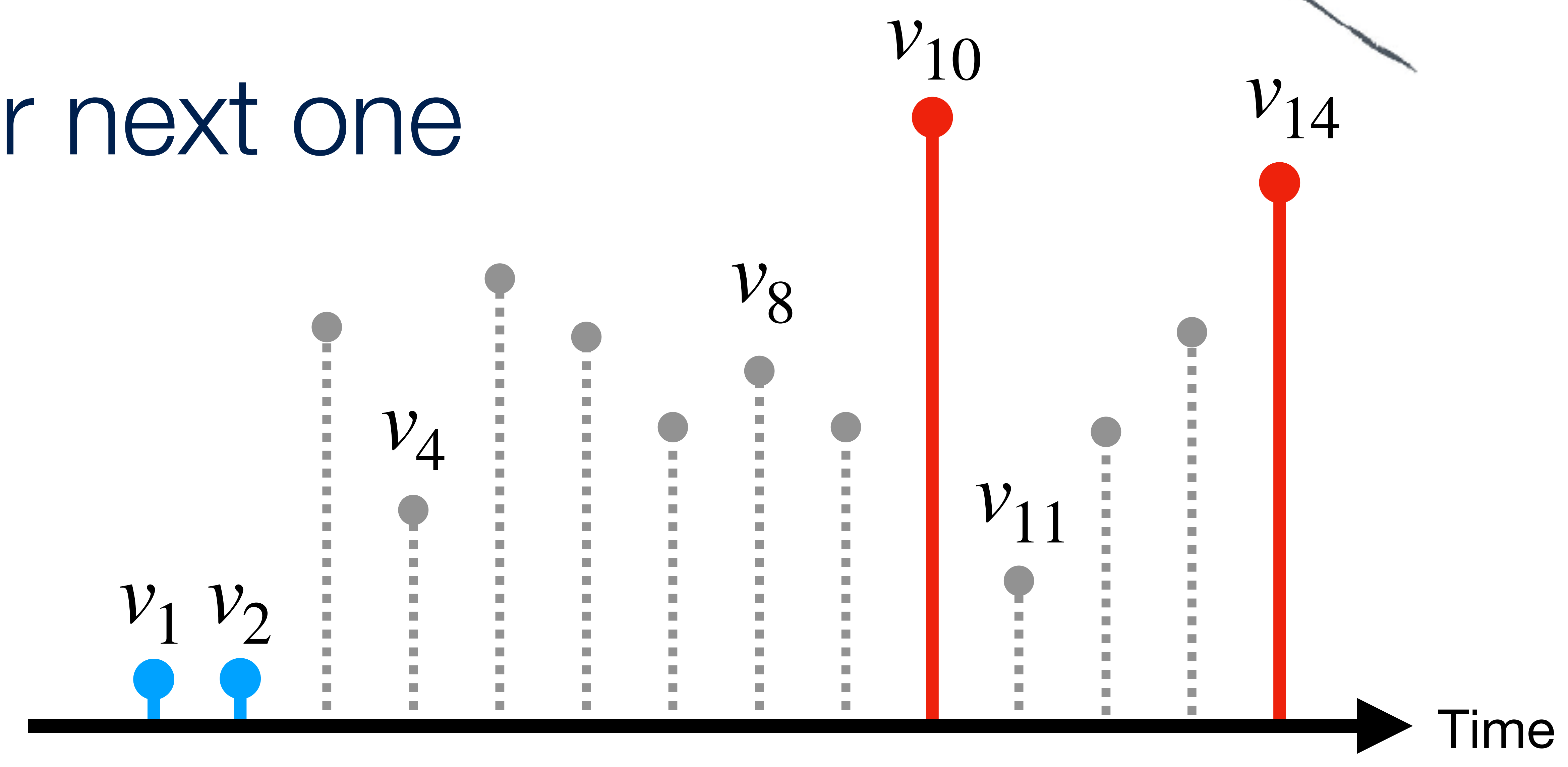
Worst-case competitive analysis

Example 2: k-max Search

- **Selling** k items for a sequence of buyers
- Each buyer $t = 1, 2, \cdots, T$ makes an **offer** v_t
- **Accept it and sell 1 item**, or wait for next one
- Once sold, cannot be reclaimed



$$\frac{\text{Profit (Opt)}}{\text{Profit (Alg)}} = \frac{v_{10} + v_{14}}{v_1 + v_2} > 1$$



e.g., $k = 2,\ T = 14$

Algorithmica 2009

Optimal Algorithms for k-Search with Application in Option Pricing

Julian Lorenz[1], Konstantinos Panagiotou[2], and Angelika Steger

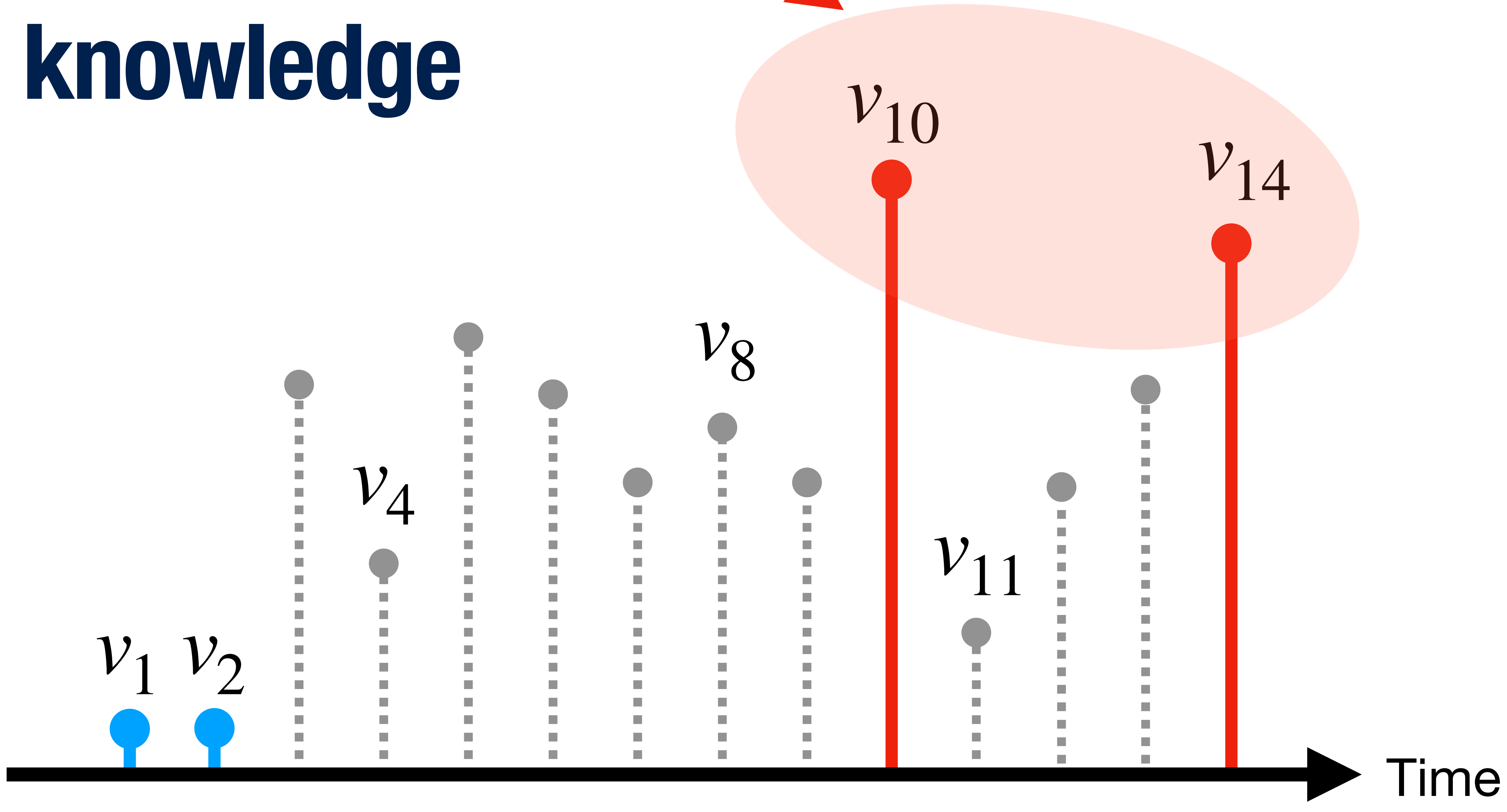
$$\max_{x_t} \sum_{t=1}^{T} v_t x_t$$

$$\text{subject to} \quad \sum_{t=1}^{T} x_t \le k,$$

$$x_t \in \{0,1\}, \forall t$$

Assume all future knowledge

$$\frac{\text{Profit (Opt)}}{\text{Profit (Alg)}} = \frac{v_{10} + v_{14}}{v_1 + v_2} > 1$$



e.g., $k = 2,\ T = 14$

Example 3: One-Way Trading

Algorithmica 2001

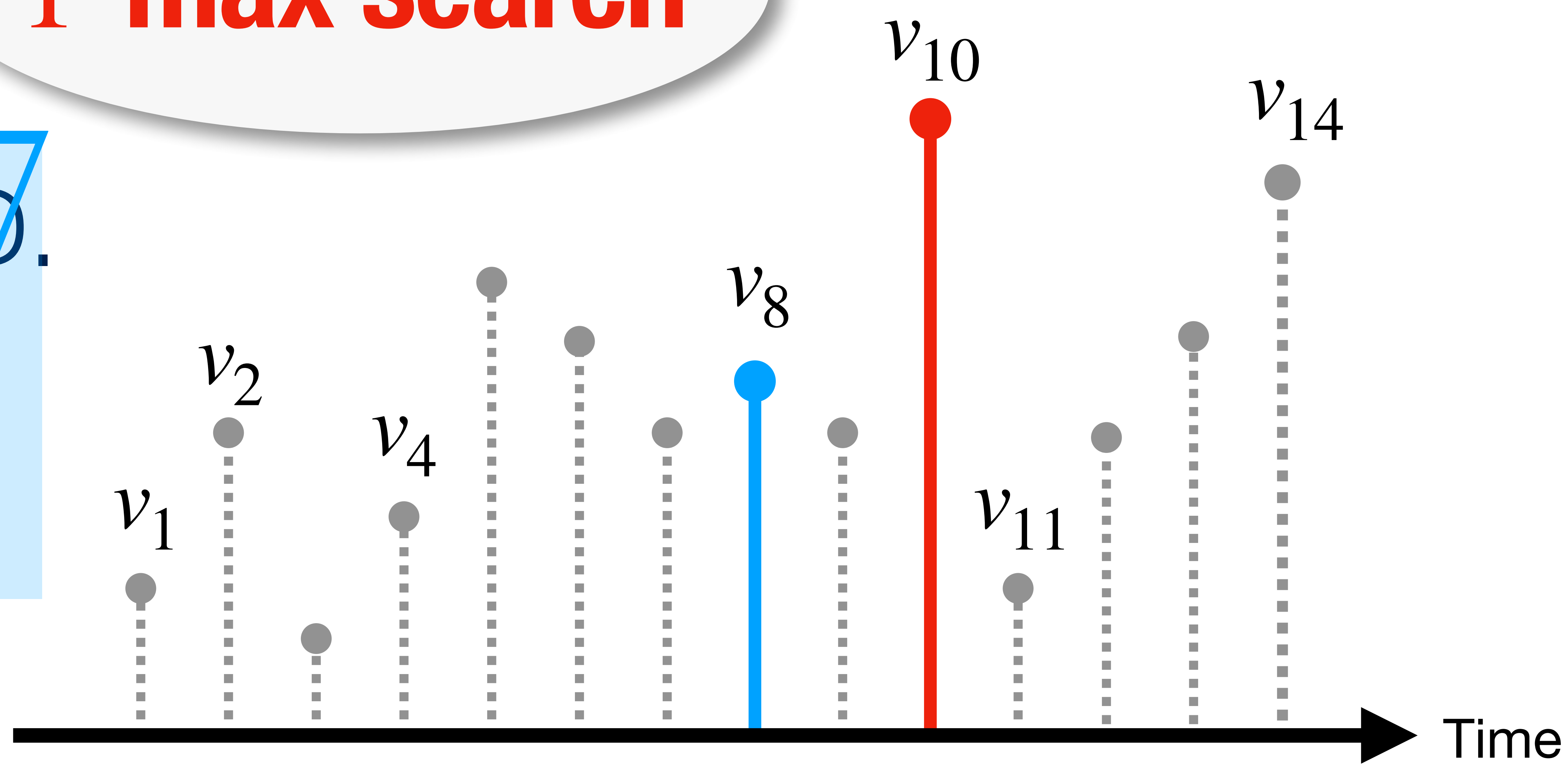
Optimal Search and One-Way Trading Online Algorithms

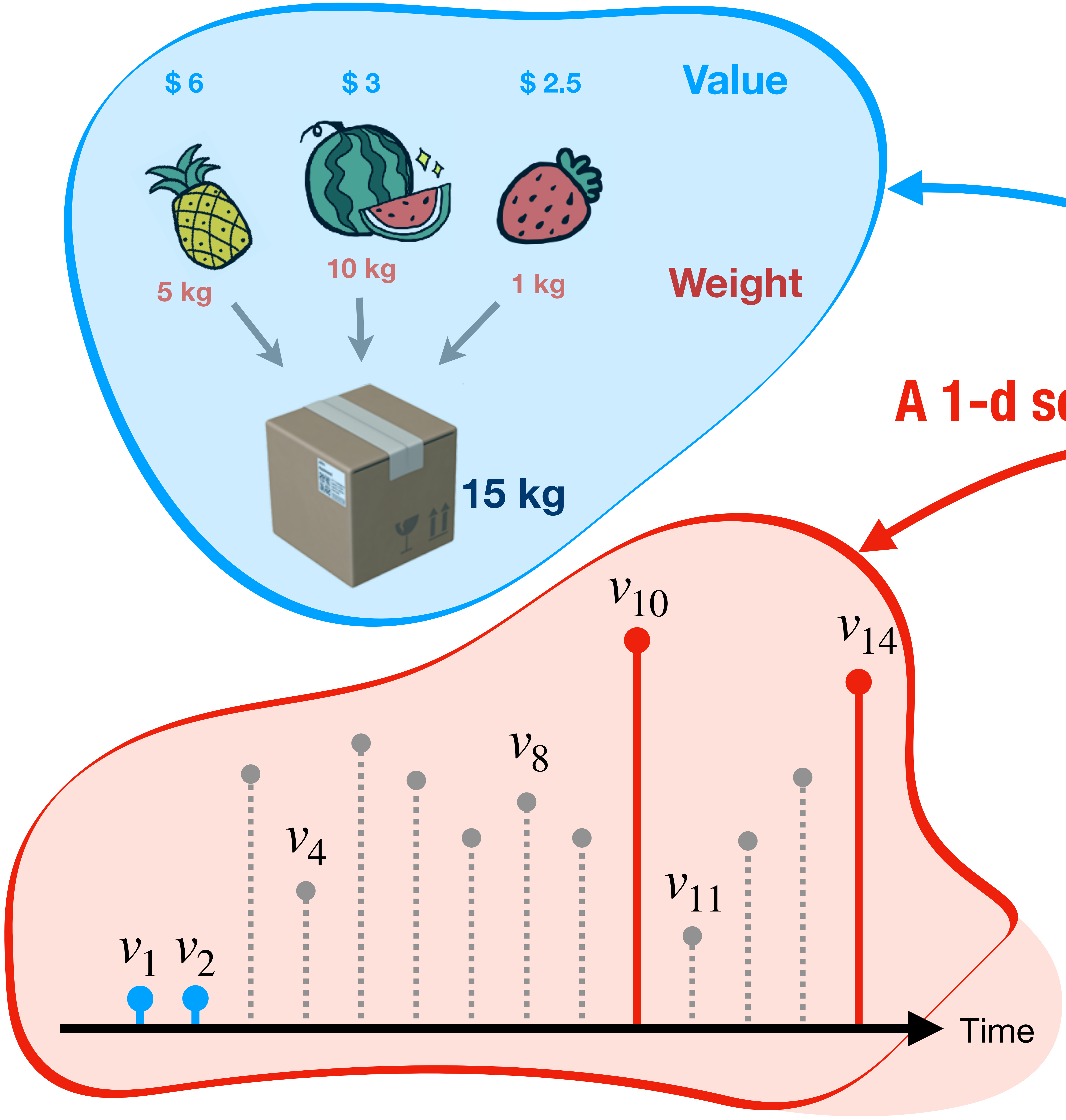
R. El-Yaniv,[1] A. Fiat,[2] R. M. Karp,[3] and G. Turpin[4]

- To change 1 CAD for USD.
- Rates v_t are revealed sequentially for $t = 1,2,\cdots,T$
- At step t: Trade x_t CAD for USD.
- **Maximize** USD in the end.

Many variants…

1**-max search**





May have multi-dimensional attributes (e.g., value and weight)

A 1-d scale value

- A set of T items (one at a time)
- Select a **subset** $S \subset T$ of items with possible **constraints**
- Maximize **objective** $v(S)$

NeurIPS 2021

Pareto-Optimal Learning-Augmented Algorithms for Online Conversion Problems

Bo Sun
ECE, HKUST
bsunaa@connect.ust.hk

Russell Lee
CICS, UMass Amherst
rclee@cs.umass.edu

Mohammad Hajiesmaili
CICS, UMass Amherst
hajiesmaili@cs.umass.edu

Adam Wierman
CMS, Caltech
adamw@caltech.edu

Danny H.K. Tsang
ECE, HKUST
eetsang@ust.hk

Online Selection Problems against Constrained Adversary

Zhihao Jiang [1] **Pinyan Lu** [2] **Zhihao Gavin Tang** [2] **Yuhao Zhang** [3]

ICML 2021

An Efficient Framework for Balancing Submodularity and Cost *

Sofia Maria Nikolakaki Alina Ene Evimaria Terzi

SIGKDD 2021

Algorithmica 2001

Optimal Search and One-Way Trading Online Algorithms

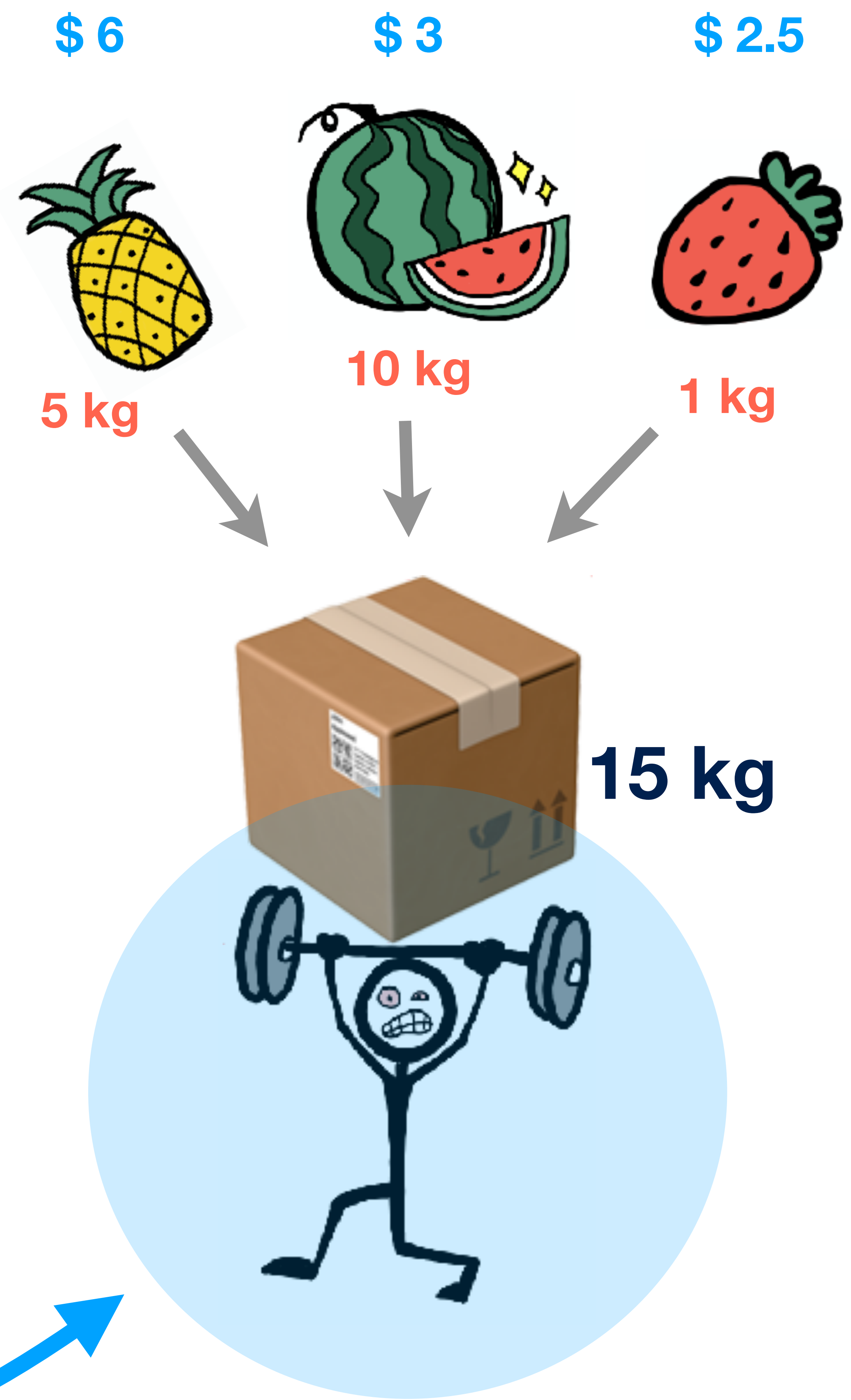
R. El-Yaniv,[1] A. Fiat,[2] R. M. Karp,[3] and G. Turpin[4]

Online selection problems

- A set of T items (one at a time)
- Select a **subset** $S \subset T$ of items with possible **constraints**
- Maximize **objective** $v(S)$

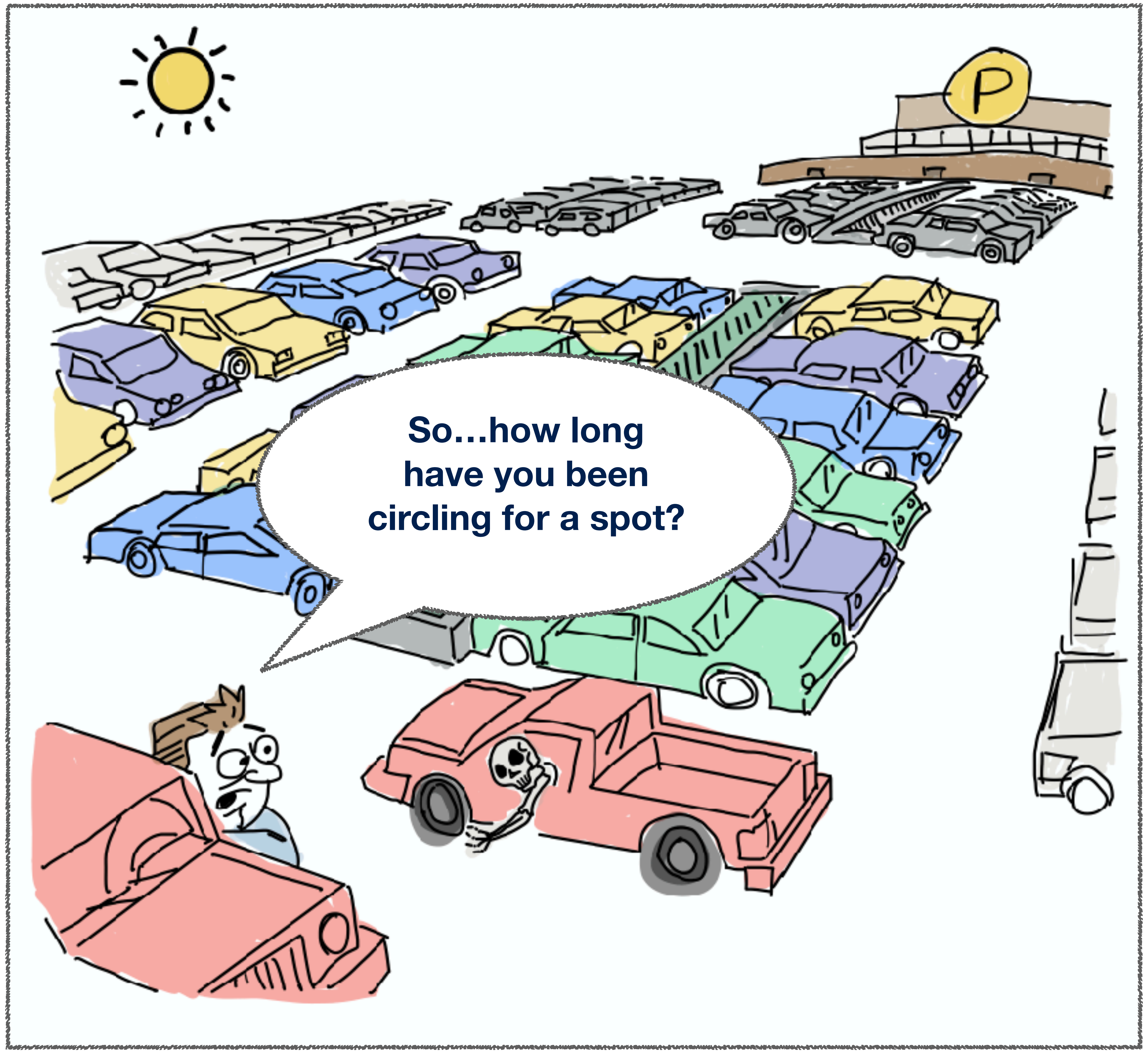
A New Variant: OSCC

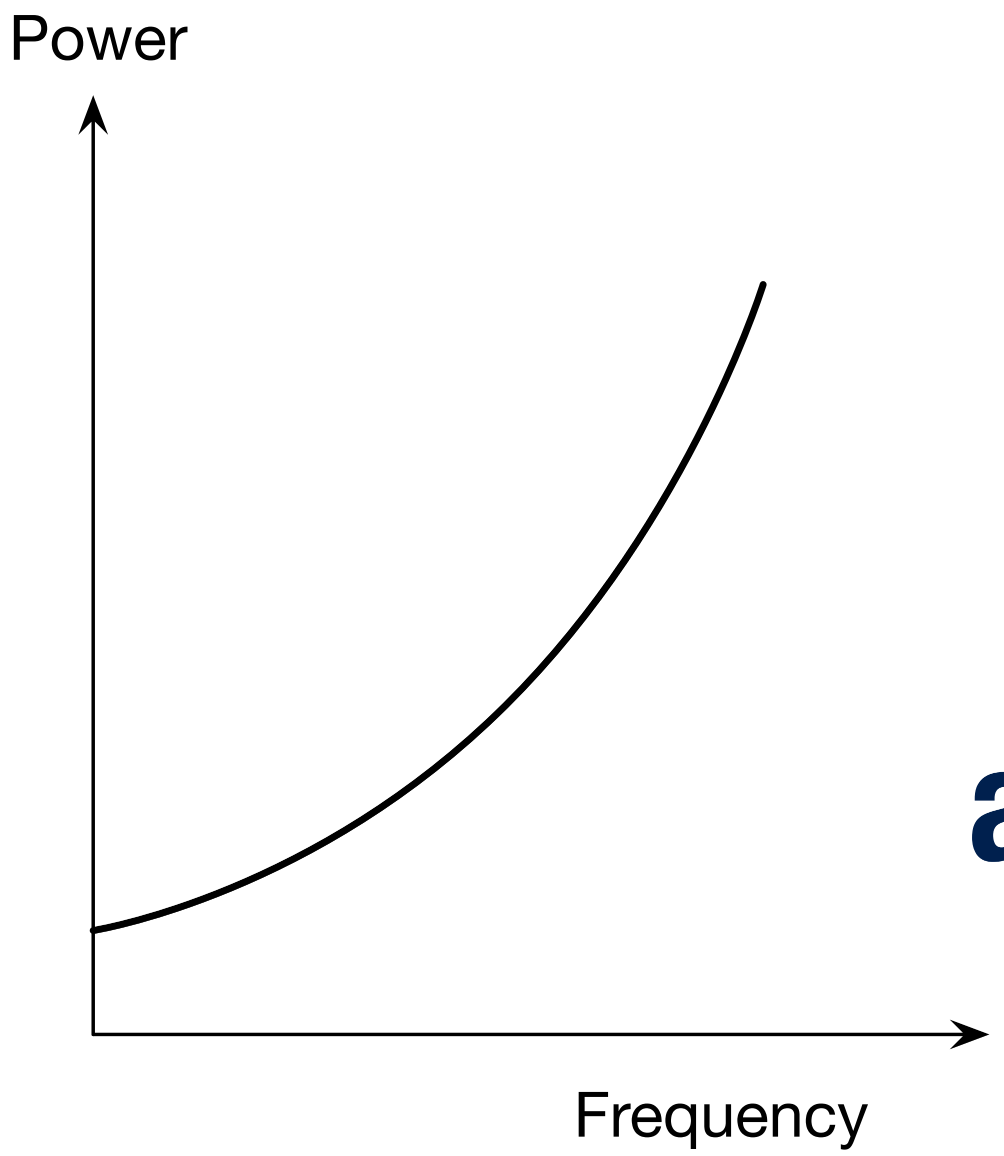
- A set of T items (one at a time)
- Select a subset $S \subset T$ of items (with possible constraints)
- Maximize objective $v(S) - f(S)$



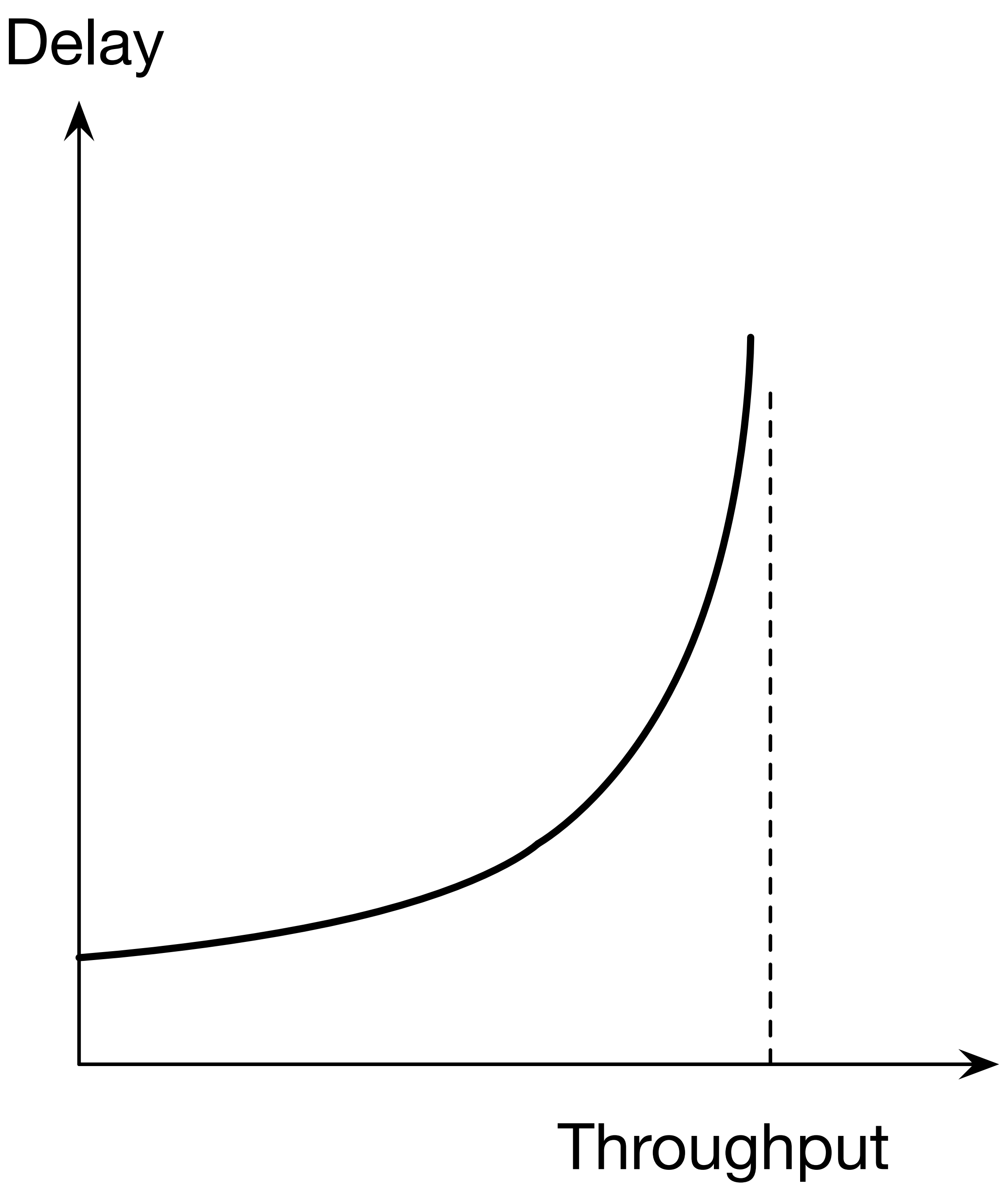
OSCC: Online selection with convex costs

Convex & increasing costs





Cloud resource allocation



Network resource allocation

OSCC: Value-Cost Tradeoff

- A set of T items (one at a time)
- Select a subset $S \subset T$ of items (with possible constraints)
- Maximize objective $v(S) - f(S)$

OSCC and More Variants

- A set of T items (one at a time) → **Multi-dimensional?**
- Select a subset $S \subset T$ of items (with possible constraints) → **Combinatorial?**
- Maximize objective $v(S) - f(S)$

$v(S)$ → **Linear? Separable?**

$f(S)$ → **Convex? Separable?**

Welfare Maximization with Production Costs: A Primal Dual Approach

Zhiyi Huang* The University of Hong Kong zhiyi@cs.hku.hk

Anthony Kim† Stanford University tonyekim@stanford.edu

Games and Economic Behavior (2018)

Mechanism Design for Online Resource Allocation: A Unified Approach

XIAOQI TAN, University of Toronto, Canada
BO SUN, HKUST, Hong Kong, China
ALBERTO LEON-GARCIA, University of Toronto, Canada
YUAN WU, University of Macau, Macau, China
DANNY H.K. TSANG, HKUST, Hong Kong, China

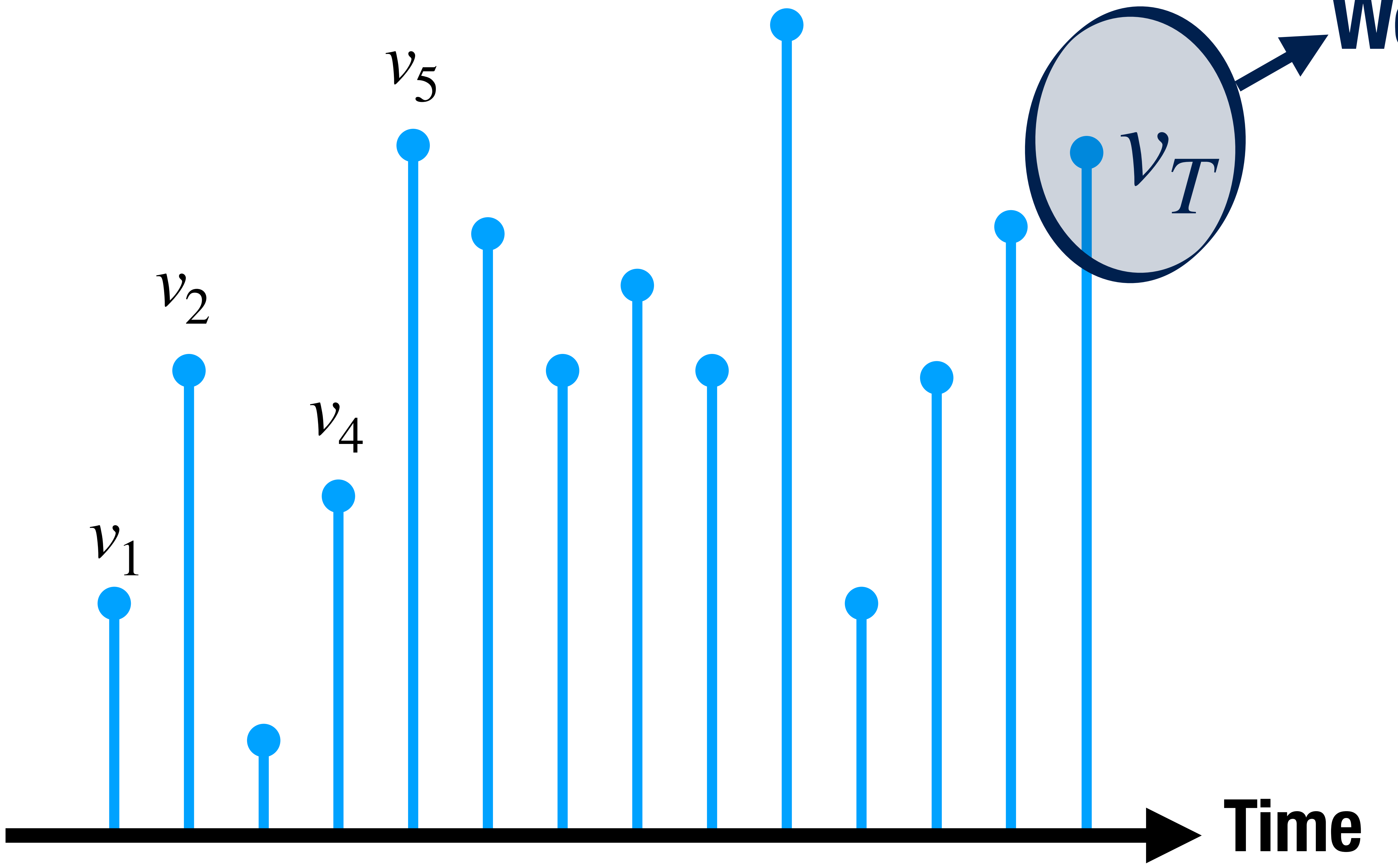
SIGMETRICS 2020

Online Combinatorial Auctions for Resource Allocation with Supply Costs and Capacity Limits

Xiaoqi Tan, Alberto Leon-Garcia, Yuan Wu, and Danny H.K. Tsang

IEEE-JSAC 2020

OSCC: Basic Setting (For Today)



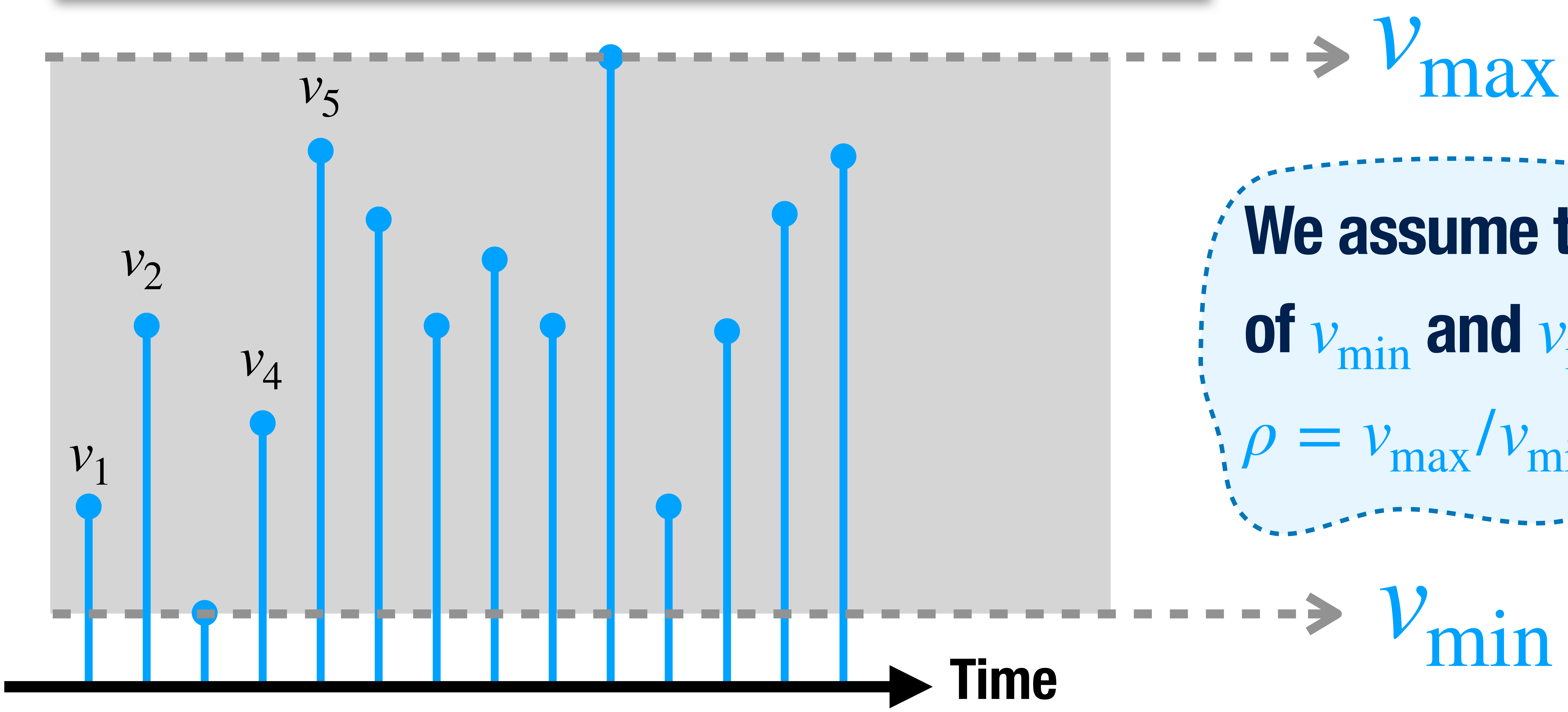
We do not know T, but assume $T \geq k$

$$\begin{aligned} \underset{\{x_t\}_{\forall t}}{\text{maximize}} \quad & \sum_{t=1}^{T} v_t x_t - f\left(\sum_{t=1}^{T} x_t\right) \\ \text{subject to} \quad & \sum_{t=1}^{T} x_t \leq k, \\ & x_t = \{0,1\}, \forall t. \end{aligned}$$

Given a priori

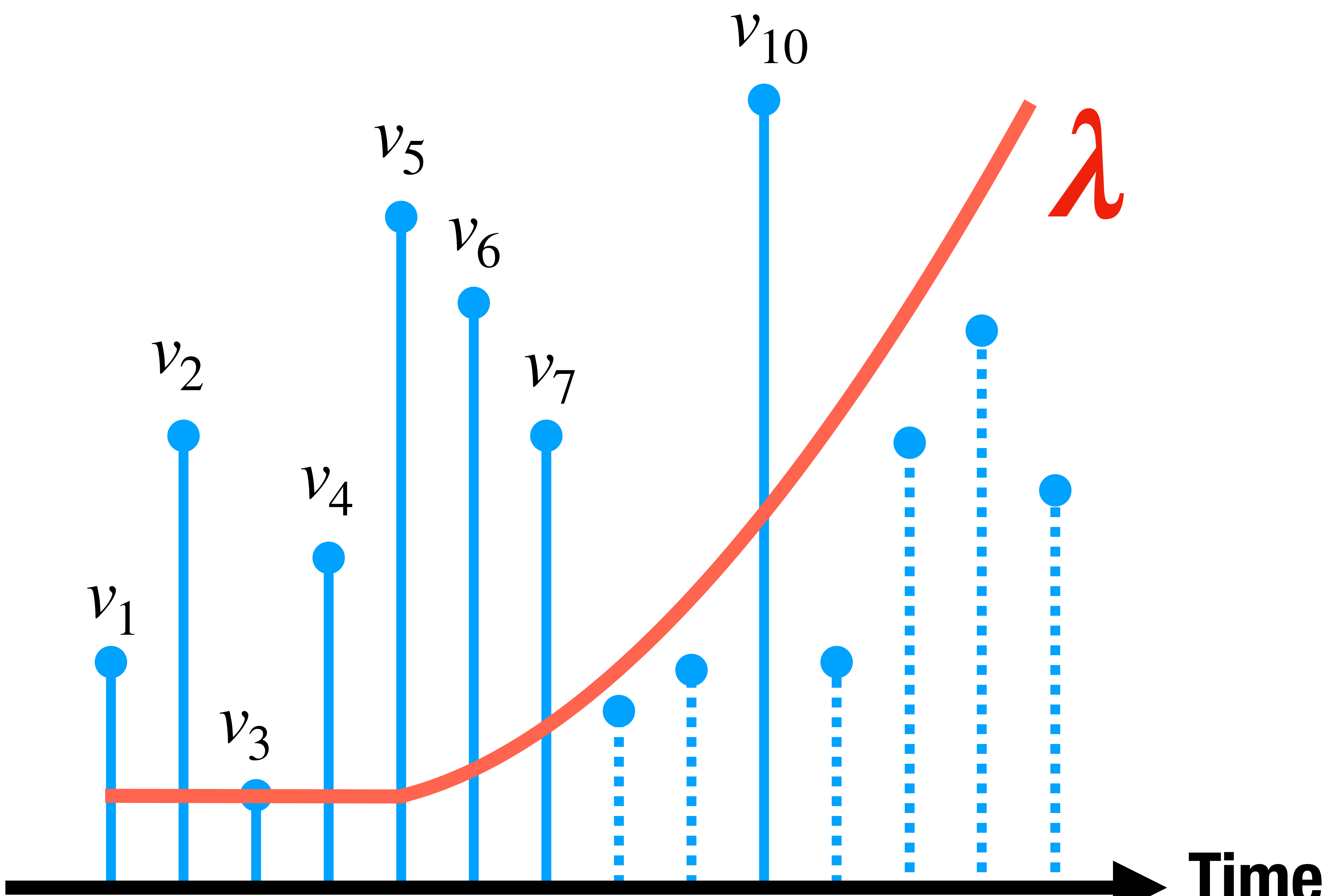
~~In this setting:~~ OSCC = k-max Search with Convex Costs

Larger $\rho \implies$ Higher levels of uncertainty



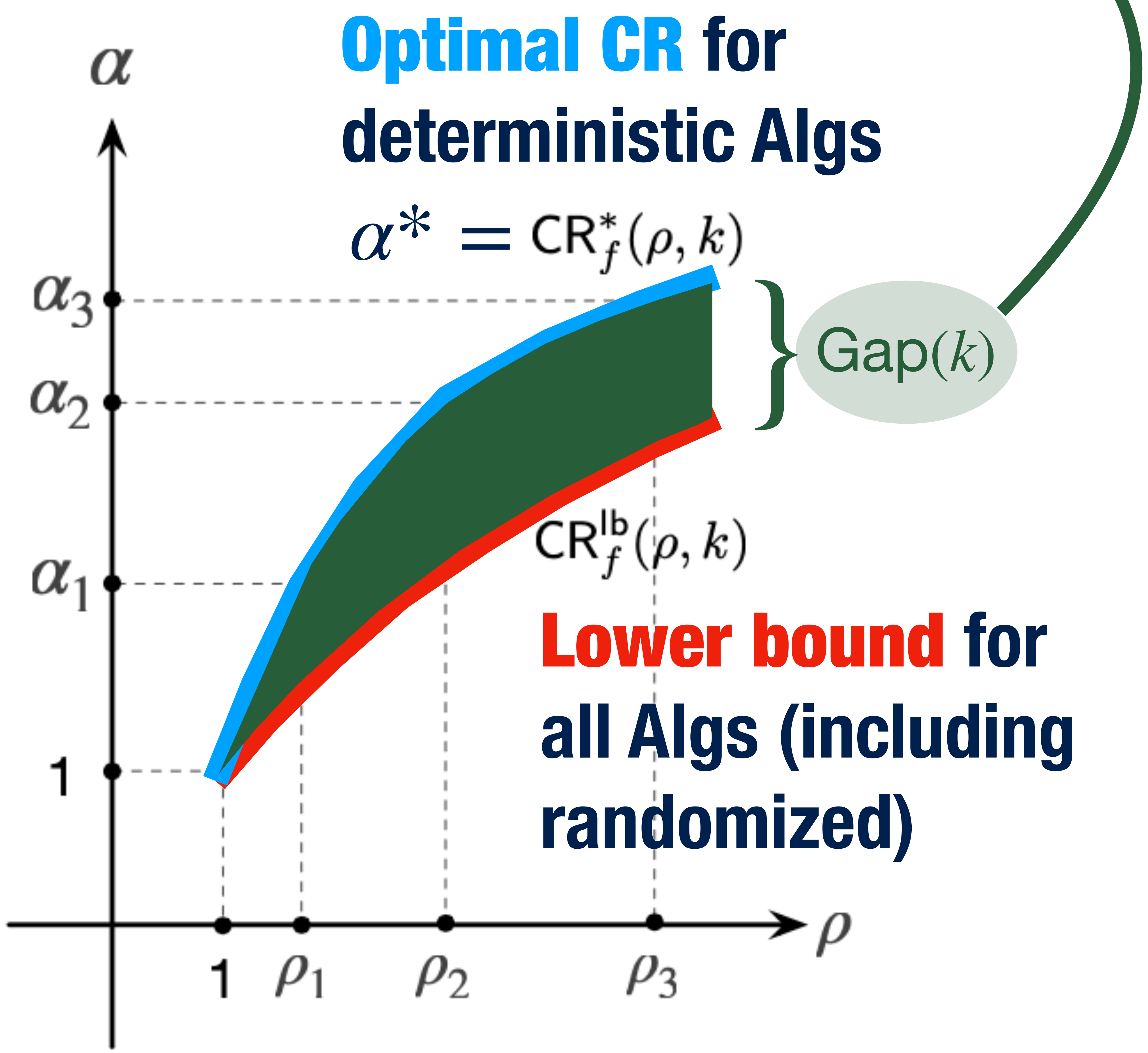
We assume the knowledge of v_{min} and v_{max}, and define $\rho = v_{max}/v_{min}$

Our Algorithm: TP_λ

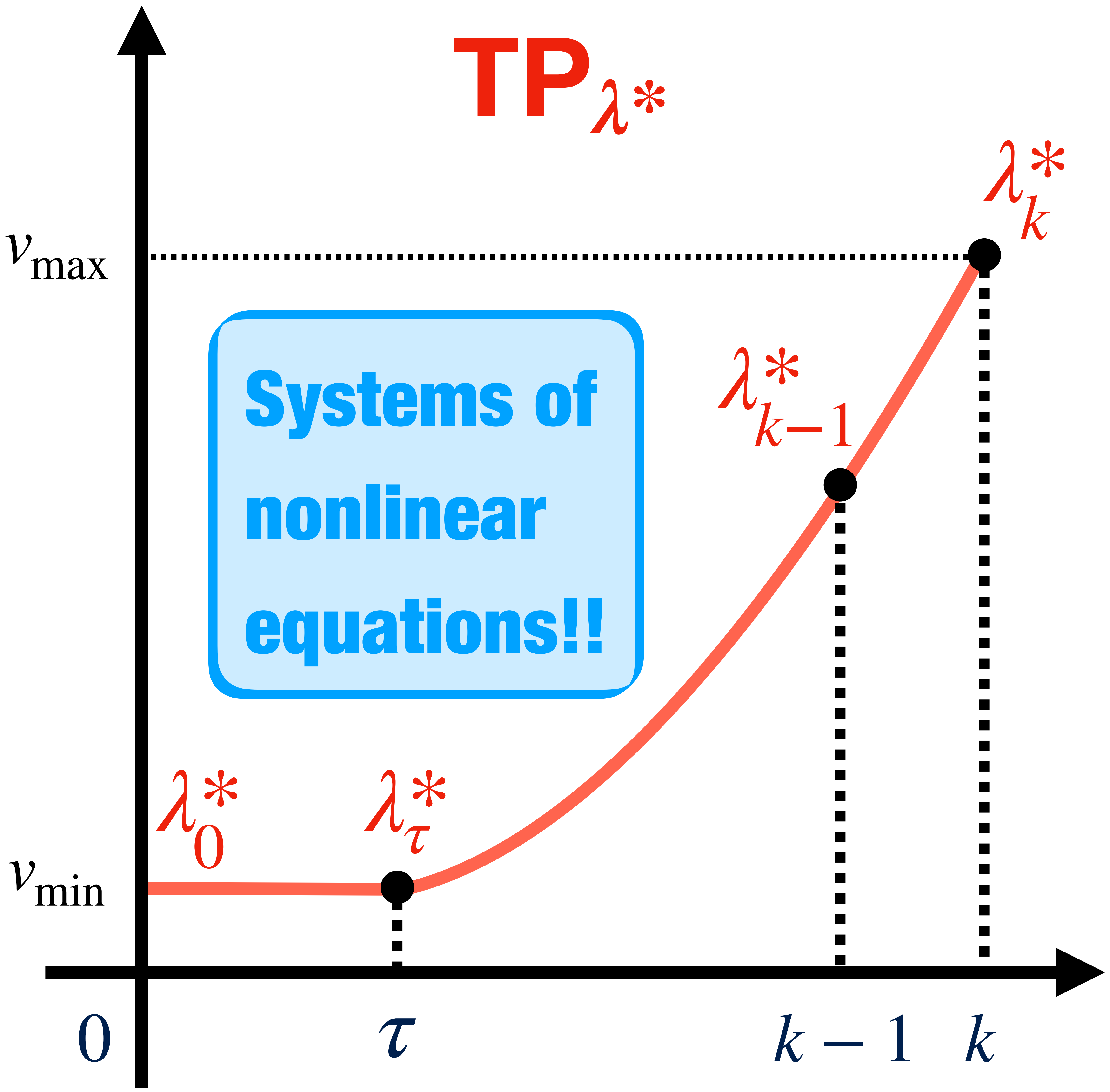


$$\rho = v_{\max}/v_{\min}$$

$$\lim_{k\to\infty} \text{Gap}(k) = 0$$



TP_{λ^*}



Theorem 1. $\mathrm{TP}_{\boldsymbol{\lambda}^*}$ *achieves the best-possible CR of all deterministic algorithms, denoted by* α^*, *if and only if* $\boldsymbol{\lambda}^* = \{\lambda_0^*, \lambda_1^*, \cdots, \lambda_\tau^*, \cdots, \lambda_k^*\}$ *is an admission threshold such that*

- $\lambda_0^* = \lambda_1^* = \cdots = \lambda_\tau^* = v_{\min}$ *and* $\lambda_k^* = v_{\max}$, *where* τ *is the minimum integer in* $\{0, 1, \cdots, k-1\}$ *such that*

$$v_{\min}(\tau+1) - f(\tau+1) \geq \frac{f^*(v_{\min})}{\alpha^*}. \qquad (5)$$

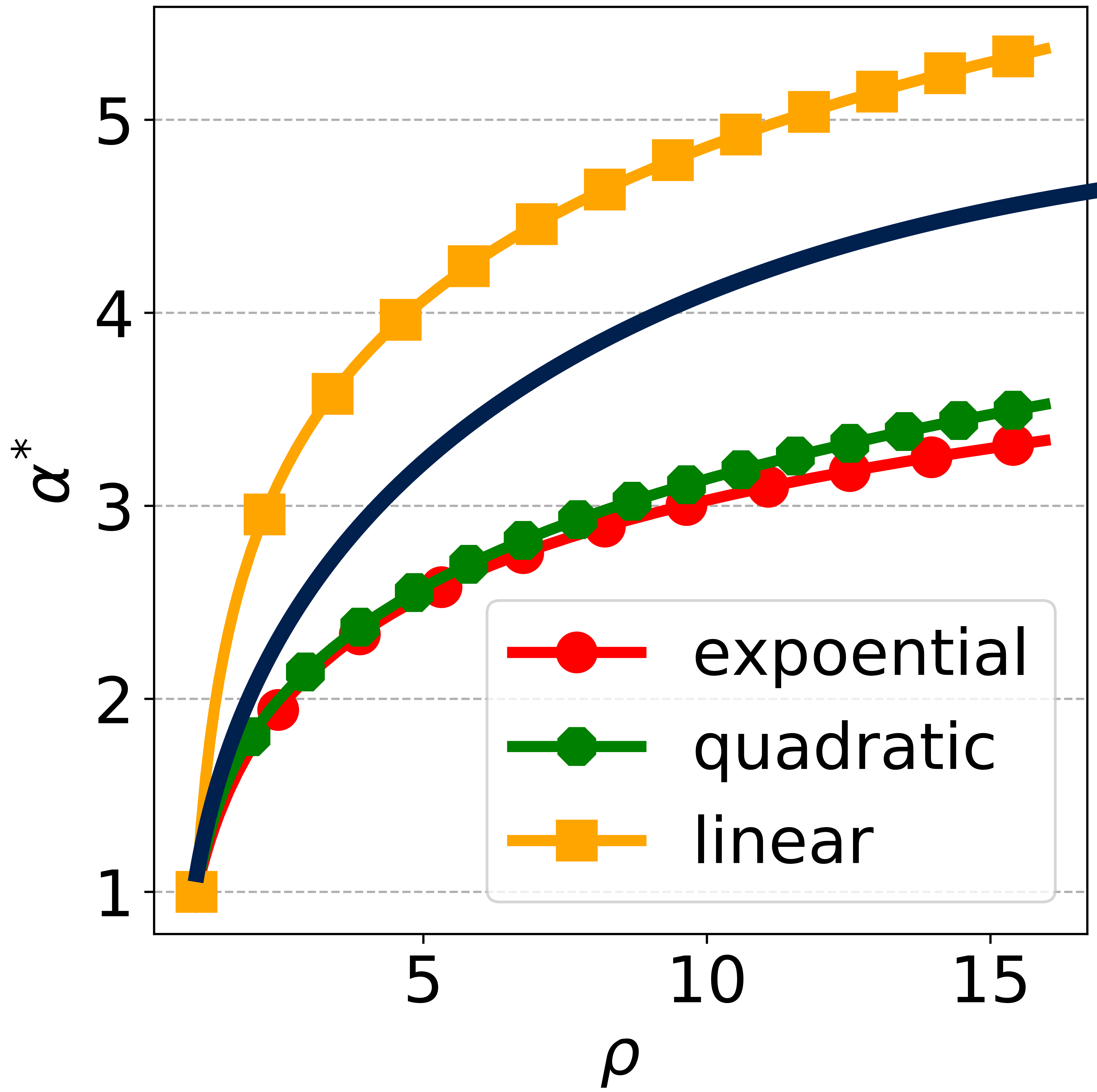
- $\{\alpha^*, \lambda_{\tau+1}^*, \lambda_{\tau+2}^*, \cdots, \lambda_{k-1}^*, \lambda_k^*\}$ *is the unique set of* $k-\tau+1$ *positive real numbers that satisfy the system of equations:*

$$\alpha^* = \frac{f^*(\lambda_{\tau+1}^*)}{v_{\min}(\tau+1) - f(\tau+1)} = \frac{f^*(\lambda_{\tau+2}^*) - f^*(\lambda_{\tau+1}^*)}{\lambda_{\tau+1}^* - c_{\tau+2}} = \cdots = \frac{f^*(\lambda_k^*) - f^*(\lambda_{k-1}^*)}{\lambda_{k-1}^* - c_k}. \qquad (6)$$

Existence of many competitive TPs

Uniqueness of optimal (deterministic) TP

Optimality among all deterministic algs



$$1 + \ln \rho$$

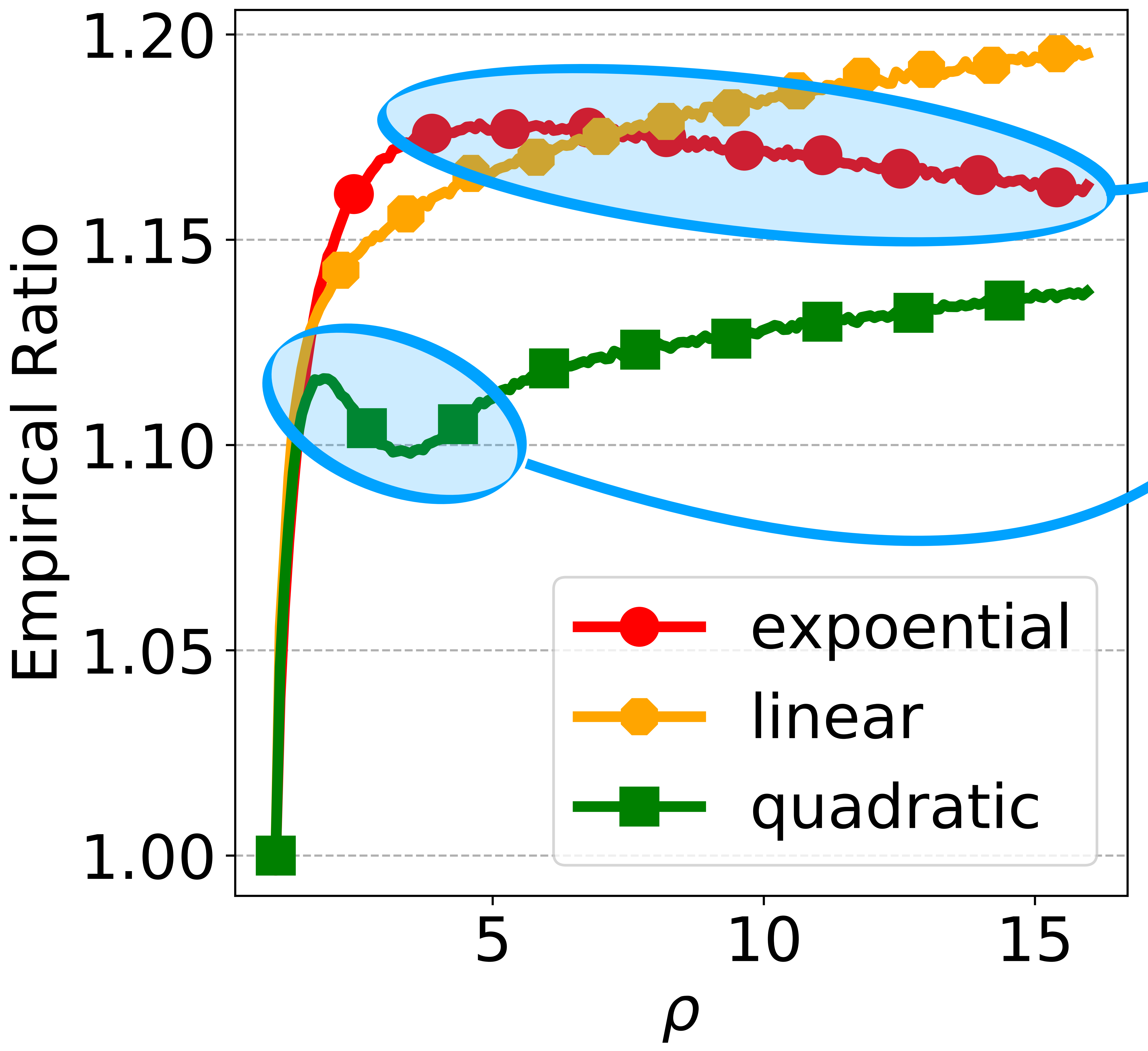
Strongly convex

Higher level of uncertainty $\Longrightarrow$ **worse performance** (larger α^*)

Faster growth rate of cost $\Longrightarrow$ **better performance** (smaller α^*)

Worst-case performance

$$\text{Empirical Ratio} \triangleq \frac{1}{N}\sum_{n=1}^{N}\frac{\mathsf{OPT}(\mathcal{I}_n)}{\mathsf{TP}_{\lambda^*}(\mathcal{I}_n)}$$



Higher level of uncertainty does not always lead to worse (empirical) average performance!!

Need for more research:

- Flaws of worst-case analysis -> **beyond worst-case??**
- Implication of different costs on **real-world systems design??**

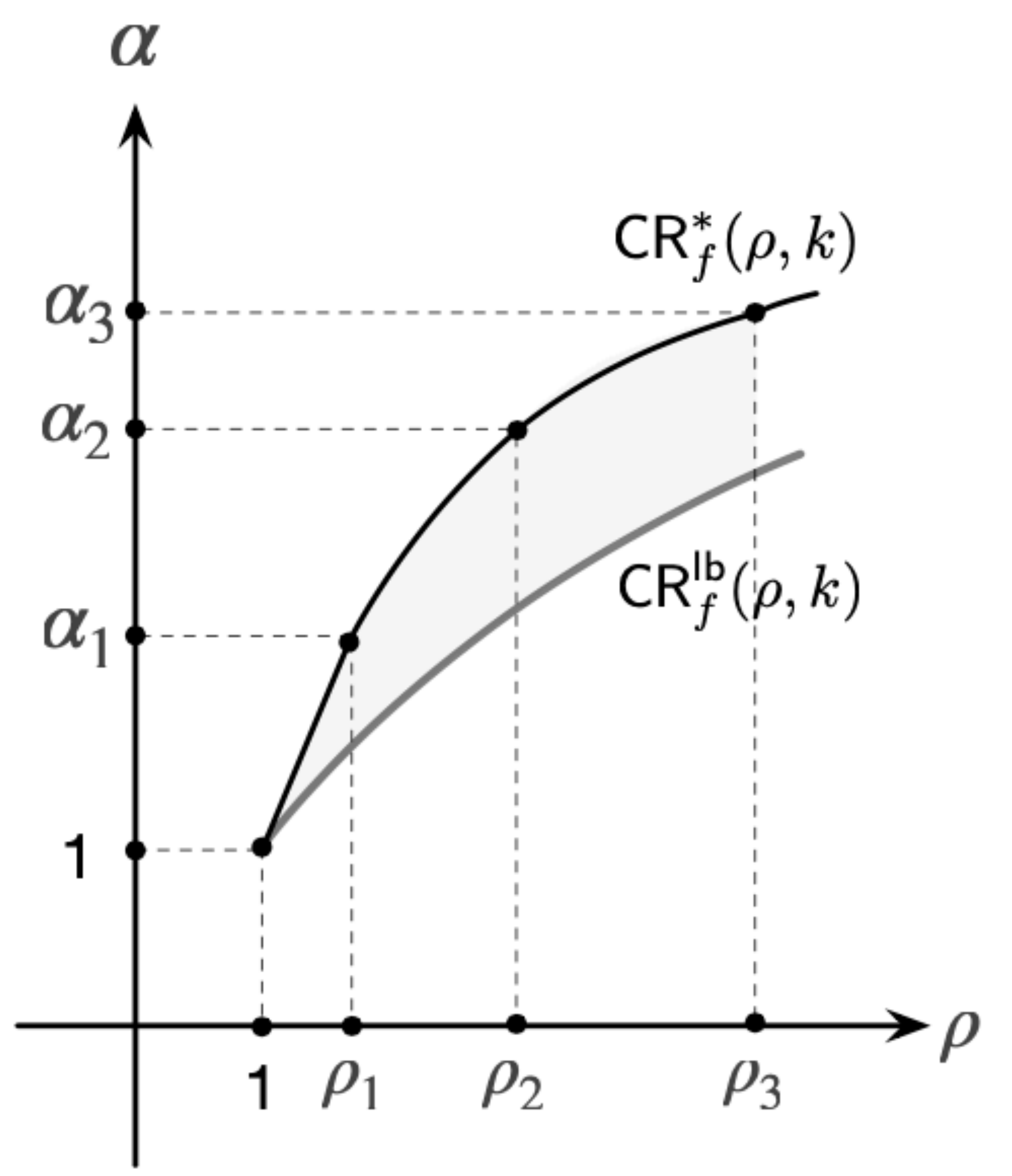
Concluding Remarks

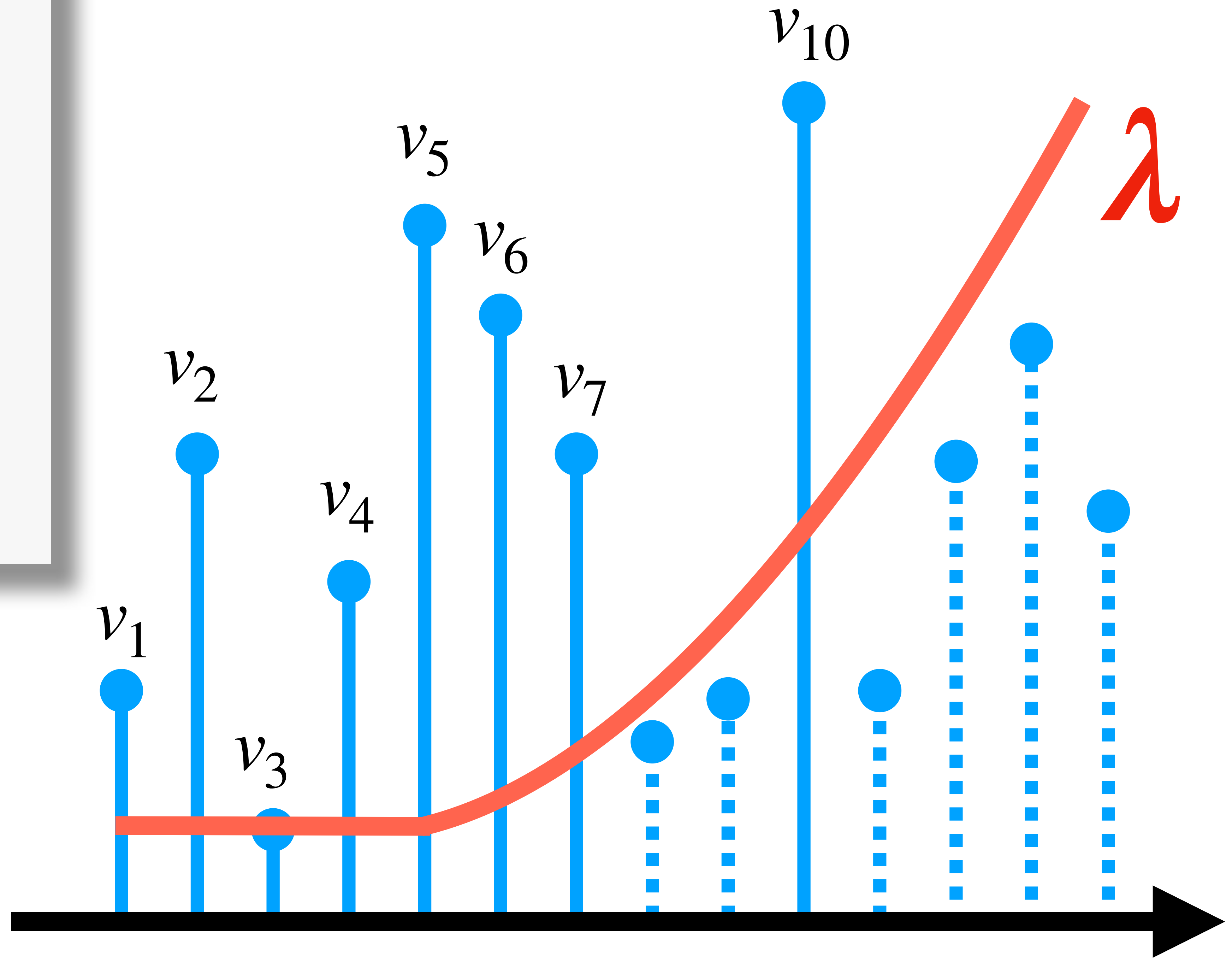
- A new variant of online optimization problem: **online selection with convex costs**

- **Simple & intuitive** algorithms, **strong** performance guarantees

- **Interesting applications** & **open questions**

Thank You





Online Selection with Convex Costs

Xiaoqi Tan[1], Siyuan Yu[1], Raouf Boutaba[2]

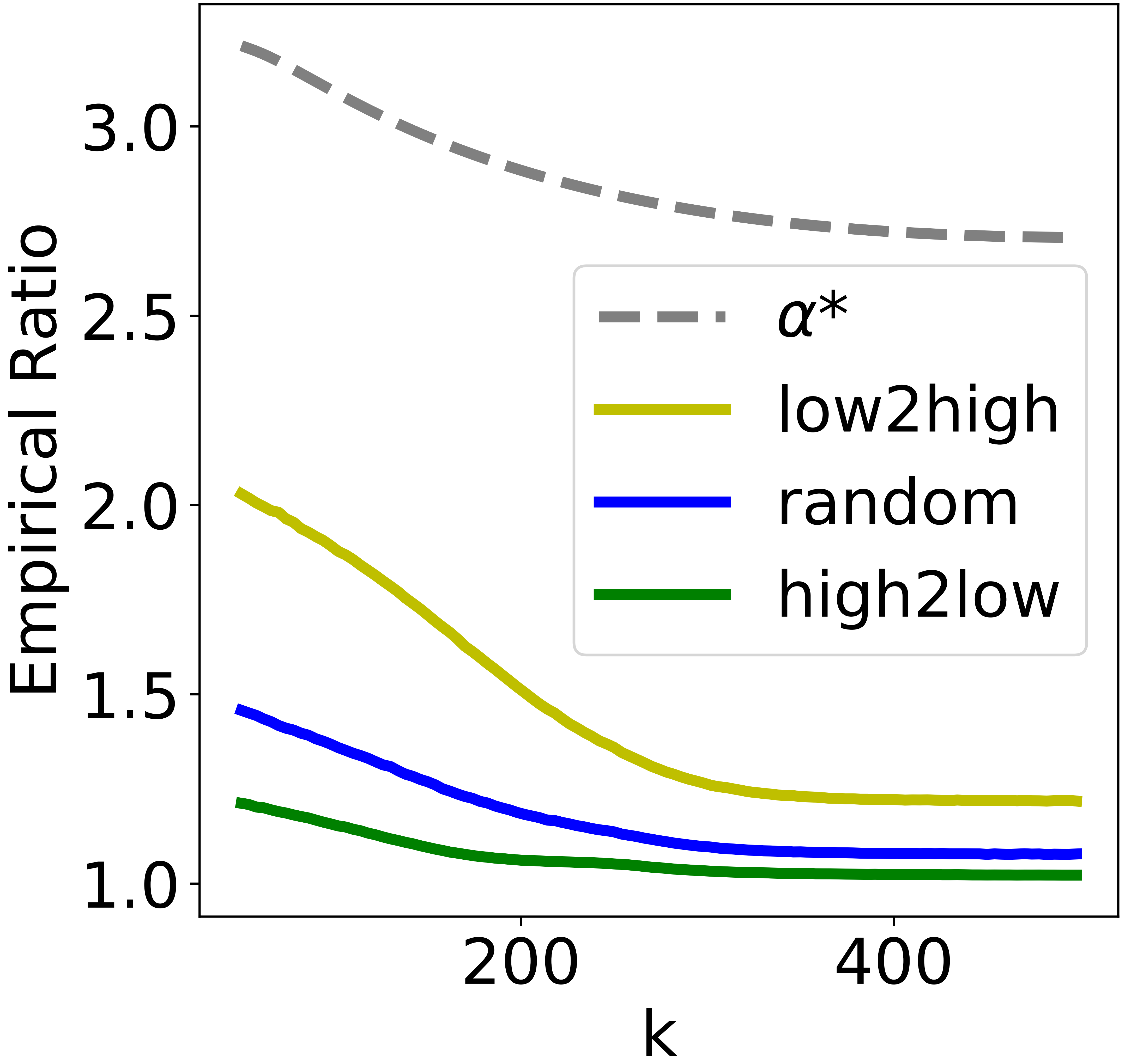
[1]University of Alberta, Canada

[2]University of Waterloo, Canada

MAMA Workshop June 6, 2022

$$\text{Empirical Ratio} \triangleq \frac{1}{N}\sum_{n=1}^{N}\frac{\mathsf{OPT}(\mathcal{I}_n)}{\mathsf{TP}_{\lambda^*}(\mathcal{I}_n)}$$

Different arrival instances



- **low2high:** 1st half (250 items) low values (randomly sampled from $[v_{\min}, (v_{\min}+v_{\max})/2]$) and 2nd half high

- **high2low:** 1st half (250 items) high values (randomly sampled from $[(v_{\min}+v_{\max})/2, v_{\max}]$) and 2nd half low

- **random:** all 500 items are randomly sampled from $[v_{\min}, v_{\max}]$