Inventory Planning and Real-Time Routing for Network of Electric Vehicle Battery-Swapping Stations

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***Abstract*— Battery-swapping stations (BSSs) are one of the main types of electric vehicle (EV) refueling facilities. By battery swapping, EVs first replace their depleted batteries (DBs) with fully charged ones, and then, the demounted DBs can be recharged in charging facilities in a stand-alone mode, leading to a decouple between batteries and EVs during refueling. This article targets the planning and operation of a network of geographically distributed BSSs, termed BSS-Net. In particular, we focus on two important decisions being made within two different timescales, namely, a long-term decision on planning the initial inventory in each individual BSS and a short-term decision on real-time vehicle-to-station (V2S) routing of EVs. We formulate a two-stage optimization problem and propose a two-step solution scheme. Specifically, in the first step, we determine the long-term initial inventory by sample average approximation, and the resulting planning decision leads to a maximized total expected revenue for the BSS-Net. Based on the optimal initial inventory, we design a randomized online algorithm in the second step to perform real-time V2S routing, without assuming any future EV arrival information. We rigorously prove that the worst case performance of the randomized online algorithm is theoretically bounded by a closed-form competitive ratio.**

***Index Terms*— Battery swapping, choice model, inventory planning, randomized online recommendation, vehicle-to-station (V2S) routing.**

NOMENCLATURE

Indices:

n	Index for customers' requests in an arrival sequence.
m	Index for battery-swapping stations (BSSs).
j	Index for price levels of each BSS.
ℓ	Index for samples in sample average approximation (SAA).

Sets:

$\mathcal{N}$	Set of EVs submitting requests to the BSS-Net.
$\mathcal{M}$	Set of BSSs.
$\mathcal{J}_m$	Set of price level indices in BSS m.
$\mathcal{R}_m$	Set of prices in BSS m.
$\mathcal{A}$	Set of all possible (station, price) recommendations.
S	Set of (station, price) combinations, $S \subseteq \mathcal{A}$.

Parameters:

N	Number of customers.
M	Number of BSSs.
J	Number of price levels in each BSS.
L_m^j	jth segment border of asymptotic value function ϕ_m.
$\tilde{L}_m^j$	jth segment border of discrete value function $\tilde{\phi}_m$.
c	Competitive ratio in the asymptotic case.
$\tilde{c}$	Competitive ratio in the general case.
r_m^j	jth smallest price in BSS m.
k_m	Initial inventory in BSS m.
α	Charging cost for one unit of fully charged battery (FB).
β_m	Cost for delivering one unit of FB to BSS m.
$\underline{k}, \bar{k}$	Minimum and maximum numbers of FBs delivered in one service horizon.
$\bar{\ell}$	Number of samples in SAA.

Variables:

$p_{n,m}^j(S)$	Probability of EV n accepting BSS m with price j given the recommendation S.
$x_n(S)$	Binary variable: "1" if the operator recommends S and "0" otherwise.
$x_n^\ell(S)$	Binary variable: the operator's recommendation decision in ℓth sample in SAA.
k_m	Decision variable: the initial inventory of BSS m.
ϕ_m	Asymptotic value function on the continuous inventory level of BSS m.
$\tilde{\phi}_m$	General (true) value function on the discrete inventory level of BSS m.
ξ	Realization of all of the uncertain information.
ξ_ℓ	ℓth sample in SAA.
$V_{n-1,m}$	Number of FBs consumed in BSS m before the arrival of EV n.
R_{on}	Total revenue earned by an online algorithm.
λ_m, μ_n	Dual variables to the primal problem.
U_n	Pseudorevenue by serving EV n.
w_m	Continuous fraction of consumed FBs in BSS m.

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γ Random seed in the randomized procedure, drawn from [0, 1].

I. Introduction

MOTIVATED by the increasing concern of environmental pollution and fossil fuel shortage, transportation electrification, namely, the process of integrating a large fleet of public and private electric vehicles (EVs) into the transportation system, is conceived to be one of the promising solutions. For this reason, various EV refueling infrastructures have been implemented and commercialized all over the world, e.g., plug-in charging stations [1], [2], and battery-swapping stations (BSSs) [3], [4]. Compared with the plug-in charging method, which usually takes hours, EVs with swappable batteries can replace their depleted batteries (DBs) with fully charged batteries (FBs) at BSSs within minutes [4]. The replaced DBs can be recharged in a centralized charging facility (CF) and redistributed to BSSs through logistics. Thus, battery swapping can work as a complementary method to plug-in charging to satisfy the EV refueling demand in an urban city. According to [4]–[6], a well-designed and well-operated network of BSSs, termed BSS-Net hereinafter, has been demonstrated to be effective in providing fast driving range extension services for EVs.

The promising potential of battery swapping has drawn increasing academic and industrial attention from both the transportation and power system domains. In particular, extensive studies have investigated the optimal charging scheduling of DBs in CFs by different modeling techniques (see [5]–[8]). In these studies, the common objective is to design charging strategies so that a certain system-wide objective can be optimized, e.g., cost minimization for the BSS-Net operator [5], [6] or social welfare maximization for both the power and transportation networks [7], [8]. However, all the aforementioned works neglect the key operational decisions in BSS-Nets, namely, the inventory planning of BSSs and the vehicle-to-station (V2S) routing of EVs. Specifically, the inventory planning of BSSs refers to the process of maintaining a certain initial inventory of FBs for each BSS at the beginning of the service time horizon (i.e., a long-term decision in days or even weeks), while the V2S routing represents the process of directing the FB demands from EVs to different BSSs in real time (i.e., a short-term decision in minutes or even seconds). Without a system-wide inventory planning and V2S routing, EV customers will randomly choose BSSs based on their own preferences, which may lead to unbalanced loads of FB requests or even FB shortage. Therefore, a well-operated BSS-Net necessitates a careful joint decision-making in two timescales, namely, the long-term inventory planning of FBs and the short-term V2S routing of EVs in real time.

In practice, the joint decision-making for BSS-Nets is nontrivial, and the key challenges originate from two types of uncertainties as follows: 1) the random arrivals of EVs for FBs in both the spatial and temporal domains and 2) the uncertainty of the EV customer accepting or rejecting the routing decisions from the BSS-Net operator. By relaxing the second type of uncertainty in some cases (e.g., for an EV fleet), the V2S assignment has been studied in [9]–[11], where the system operator owns all the EVs and directly assigns each EV to a refueling station such that a particular system-wide performance can be achieved. However, in practice, most private EVs and even some public EVs (e.g., electric taxis) are usually not committed to a BSS-Net operator and thus may lack incentives to cooperate [12]. Therefore, it is interesting yet more practical to design a V2S routing framework by recommendation with a probabilistic model of EVs' decision-making, where each EV selects refueling stations in a probabilistic manner based on both the operator's recommendation and her own preference, rather than assignment.[1]

Toward this end, this article aims to tackle the aforementioned challenges and focuses on making the joint decisions of inventory planning and recommendation-based V2S routing in BSS-Nets. In particular, we formulate a two-stage optimization problem and propose a two-step solution scheme, where the initial inventory is optimally determined in the first-step that maximizes the expected long-term revenue of the BSS-Net. In addition, we consider the scenario where FBs are only redistributed from CFs to BSSs at the beginning of the service time horizon (e.g., a day), indicating that there is no replenishment during the entire service time horizon. Based on the optimal initial inventory obtained in the first step, we propose a randomized online recommendation-based V2S routing algorithm in the second step, without assuming any future information (e.g., EV arrival sequence). Leveraging a principled primal–dual analysis, we rigorously prove the worst case performance is theoretically bounded by a closed-form competitive ratio in a general case, which greatly extends our previous results derived in the asymptotic case [13]. The detailed design and analysis of our algorithm will be elaborated in Section IV-D. Before introducing the complete two-step decision-making scheme, we next present the related work.

A. Related Work

In recent years, extensive studies have focused on the optimal charging scheduling of batteries in BSS-Nets, as mentioned earlier. For instance, Tan *et al.* [5] investigated the battery charging schedule in the CF of the BSS-Net to minimize the total charging cost while fulfilling the FB demand from BSSs. Specifically, they develop a generalized Benders decomposition algorithm to solve the charging scheduling problem efficiently. With an objective to minimize the total charging cost in BSS-Nets and meanwhile reduce the power loss in the underlying power networks, Kang *et al.* [6] presented a centralized charging scheduling strategy in a centralized CF by considering the spot electricity prices. Similar results have also been reported in [7], in which a coordinated charging scheduling strategy for all the BSSs in a BSS-Net is proposed so as to mitigate the negative impact of uncoordinated charging on power distribution networks. In addition, Widrick *et al.* [8] described the battery charging and discharging scheduling problem in BSSs by utilizing the Markov decision process, which aims to maximize the total expected revenue of the BSS-Net over a fixed time horizon.

[1] It is worth pointing out that the recommendation-based routing includes the assignment-based routing as a special case when the probability of selecting the recommended refueling station is 1.

In comparison to charging scheduling in the CF of BSS-Nets, inventory planning of FBs in BSSs is less studied in the literature. Nie *et al.* [14] addressed the battery inventory management problem among BSSs in the BSS-Net to satisfy battery demand with the purpose of maximizing the total revenue, where a binomial distribution of battery demand is assumed. More recently, a periodic fluid model is proposed in [15] to jointly solve the BSS battery inventory planning and charging scheduling problems so as to minimize the total operational cost under stochastic battery demand and electricity prices. Beyond these works, some studies are performed to determine the initial inventory of mutually substitutable products in a general supplier–customer system, which exactly captures the nature of BSS-Nets. For instance, with the assumption of Poisson demand arrival rate, Xu *et al.* [16] investigated the problem of selling two mutually substitutable products over a fixed time horizon, where both the selling rules and initial product inventory are made to maximize the total revenue. Similarly, Yao *et al.* [17] focused on the initial inventory planning problem of a single type of product, which can be transshipped among different supply companies, and made joint decisions on the inventory planning and product transshipment. Different from the abovementioned literature, we do not assume any distribution of battery demand and formulate the initial FB inventory planning problem into a two-stage stochastic programming problem [18], [19], with the consideration of both system revenue and customers' personal preference. Based on the hidden concavity of the formulation, we can equivalently transform the two-stage stochastic programming problem into a single-stage problem and solve it efficiently.

Other than the charging scheduling and inventory planning, the real-time V2S routing of EVs is also a key process during the operation of BSS-Nets. Most of the existing works contribute to V2S routing via assignment, by which the vehicles are assumed to follow exactly the assignment decisions. For instance, You *et al.* [9], [10] investigated the V2S assignment problem for EV battery swapping and obtained both the centralized and distributed assignment solutions with and without global information of BSS-Nets. Specifically, in [9], a centralized solution is obtained by leveraging the global information of a BSS-Net, such that the operator can assign each EV to a particular BSS and minimize a weighted sum of EVs' traveling distance and electricity cost. On the contrary, two decentralized solutions are proposed in [10] for the cases without global information of BSS-Nets. In addition, Fanti *et al.* [11] targeted the V2S assignment for a group of EVs to minimize the total charging time. Different from these works, it is also possible to perform V2S routing from a more practical viewpoint by implementing real-time V2S recommendations with the consideration of customers' choice preferences. For example, Guo *et al.* [20] investigated the EV spatial scheduling problem by recommending charging stations to EV customers by considering future EV arrivals and departures to minimize the total waiting and queuing time. In addition, Tian *et al.* [21] provided real-time recommendations of charging stations to individual electric taxi to minimize the total waiting time. However, the V2S routing algorithm in

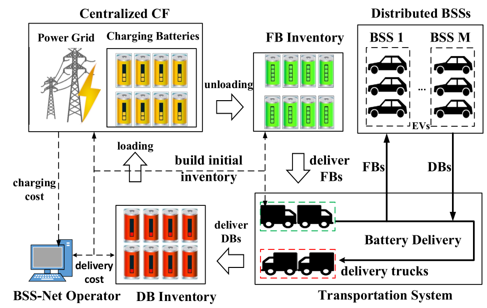

Fig. 1. Network model of the BSS-Net in an urban area.

these works is designed specifically for the plug-in charging mode. The topic on real-time V2S recommendation for the battery-swapping mode has rarely been investigated in the literature. In our previous work [13], we investigate the online V2S recommendation for battery swapping based on predetermined initial FB inventories in an asymptotic case, in which we relax the discrete inventory level of FBs to a continuous variable, such that an asymptotic performance metric is obtained by solving an ordinary differential equation. In this article, we first optimally obtain the long-term inventory of FBs with the purpose of maximizing the expected system revenue. Then, we extend the theoretical results in [13] by considering discrete inventory levels, which is a more practical and general case. By designing a randomized online V2S recommendation algorithm, a closed-form competitive ratio is rigorously proved.

B. Our Contribution

The main contributions of this article can be summarized as follows. First, we address the operation of BSS-Nets by jointly optimizing the long-term inventory planning and the short-term V2S routing, which differentiates our work from the aforementioned EV charging literature. Second, with the consideration of customers' personal choice, we formulate a two-stage revenue maximization problem to determine the long-term initial inventory in each BSS, which is highly nontrivial without assuming any demand distribution. Nevertheless, we optimally solve the initial inventory by equivalently transforming the two-stage problem into a single-stage problem based on the proven concavity of the formulation. Third, we design a randomized online V2S recommendation algorithm based on the optimal initial inventory. Without relaxing the discrete inventory level to a continuous variable in our previous work, we rigorously prove a closed-form competitive ratio in a general case. To the best of our knowledge, the proposed algorithm is the first to achieve optimal initial inventory and closed-form competitive ratio simultaneously in a general BSS-Net.

II. System Model and Problem Statement

In this section, we present the detailed BSS-Net model and describe the joint decisions on inventory planning and real-time V2S routing in the BSS-Net.

A. Network Model

We consider a BSS-Net in an urban area as shown in Fig. 1, where the BSS-Net operator owns a group of BSSs providing battery-swapping services to a population of EV customers, who can adopt battery swapping as their refueling method. Let $\mathcal{M} \triangleq \{1, \ldots, M\}$ denote the set of BSSs, which are geographically distributed in different locations. The set of EVs that submit battery-swapping requests to the BSS-Net is denoted by $\mathcal{N} \triangleq \{1, \ldots, N\}$, which are indexed based on their arrival sequences. The operator jointly makes decisions on planning the initial inventory in BSSs and the real-time V2S routing, serving EVs in a fixed service time horizon (e.g., one day) to maximize the expected total revenue.

B. Inventory Planning

We denote the initial FB inventory of BSS $m \in \mathcal{M}$ by k_m, which needs to be determined before starting to provide battery-swapping services. As shown in Fig. 1, the FBs are delivered from a centralized CF, which collects DBs from all BSSs and fully charges them for the next dispatch, with both charging and delivery costs. We consider that FBs are delivered to BSSs only at the beginning of the service time horizon (e.g., the night before the next day), namely no replenishment. After preparing the initial inventories, each BSS can provide battery-swapping service to EVs with different price levels. In particular, let $\mathcal{R}_m \triangleq \{r_m^1, \ldots, r_m^{J_m}\}$ be the possible price levels of BSS m and $\mathcal{J}_m \triangleq \{1, \ldots, J_m\}$ be the index set of price levels. Without loss of generality, the different levels of prices are ranked from low to high with the increase of indexes.

C. Online V2S Routing

We assume that the BSS-Net operator is capable of communicating with all the EVs and aims to maximize the total revenue of the BSS-Net by strategically performing the V2S routing decisions in real time. The interactions between the operator and the EVs are as follows.

1) *Request*: EV $n \in \mathcal{N}$ submits a battery-swapping request to the BSS-Net operator in real time. While submitting the request, each EV has its unique identity information (e.g., time, location, and customer ID), which can be identified by the operator.
2) *Recommendation*: Upon receiving the request from EV n, the BSS-Net operator estimates the EV's choice probability $p_{n,m}^j(S), \forall S \subseteq \mathcal{A}$. The process of obtaining $p_{n,m}^j(S)$ will be explained in detail by the case study in Section V. After that, the operator makes V2S recommendation to EV n in a complete online fashion without assuming any future information (e.g., EV arrival sequence and the total number of EVs). Based on the current EV's choice probability and the system state (i.e., the remaining FBs at all the BSSs), the BSS-Net operator aims to maximize the expected revenue by recommending an appropriate subset $S \subseteq \mathcal{A}$ of (station, price) combinations to each EV n. Note that the operator is supposed to respond to customers' requests immediately based on the arrival sequence of requests. However, the response to a customer's request can be delayed when multiple requests appear in a very short time. In this case, the operator needs to wait a few seconds for all previous customers' choices on their recommendations and consider no response as a rejection decision. When the total number of requests becomes very large, we consider the scenario that the operator responds to a request with a fixed deadline. This scenario is discussed in detail in Section V-G.
3) *Choice-Making*: After receiving the recommendation S, EV n selects one of the combinations $(m, j) \in S$ with the probability of $p_{n,m}^j(S)$ within a time window (several seconds). In particular, if EV n selects the combination (m, j), the inventory level in BSS m decreases by 1, and the BSS-Net earns a corresponding revenue r_m^j. Otherwise, EV n is considered to reject all the recommendations and chooses alternative refueling methods. Here, the choice-making of each customer (i.e., EVs in our context) in a probabilistic manner is common in the revenue management literature [22]–[25]. According to these existing studies, we assume that the choice probability $p_{n,m}^j(S)$ can be obtained from an online contextual learning process based on big data analytics, and the detailed learning process is beyond the scope of this article. In the present work, we assume that the EVs' choice probability is a given priori. Note that the basic setting for the part of real-time V2S routing is similar to [13]; however, we extend the theoretical results in a more general case, which will be explained in detail in Section IV-D.

III. Initial Inventory Planning

In this section, we focus on the initial inventory planning with the objective to maximize the expected total revenue of the BSS-Net.

A. Offline Formulation for V2S Recommendation Problem

We consider the BSS-Net operator as an independent refueling service provider, which does not own or manage the EV fleet. The ultimate goal of the operator is to maximize its expected total revenue before the next replenishment of FBs. Therefore, we start from formulating an offline revenue maximization problem by assuming the knowledge of all future uncertainties of EV arrivals. Here, a random realization of the uncertainties is summarized in $\xi \triangleq \{\mathcal{N}, \{p_{n,m}^j(S)\}_{S,n,(m,j)}\}$, including all EVs' arrival sequence $\mathcal{N}$ and the probability $p_{n,m}^j(S)$ for any EV n to accept the combination (m, j) given the recommendation set S.

Given any initial inventory k_m in each BSS m, the offline revenue maximization problem is formulated as follows:

$$R(\boldsymbol{k}, \xi) \triangleq \max_{x_n(S)} \sum_{n \in \mathcal{N}} \sum_{S \subseteq \mathcal{A}} x_n(S) \sum_{(m,j) \in S} r_m^j p_{n,m}^j(S) \tag{1a}$$

$$\text{s.t.} \sum_{n \in \mathcal{N}} \sum_{S \subseteq \mathcal{A}} x_n(S) \sum_{j \in S} p_{n,m}^j(S) \leq k_m \quad \forall m \in \mathcal{M} \tag{1b}$$

$$\sum_{S\subseteq\mathcal{A}} x_n(S) \le 1 \quad \forall n \in \mathcal{N} \tag{1c}$$

$$x_n(S) \ge 0 \quad \forall n \in \mathcal{N}\ \forall S \subseteq \mathcal{A} \tag{1d}$$

where $x_n(S)$ is the decision variable representing the probability that the operator recommends S to EV n based on the current inventory level. Note that in the objective, $\sum_{(m,j)\in S} r_m^j p_{n,m}^j(S)$ is the expected revenue by recommending $S \subseteq \mathcal{A}$ to EV n. Constraint (1b) restricts the number of consumed FBs in each BSS by its initial FB inventory. Constraint (1c) further restricts that the sum of probabilities of making different recommendations is no greater than 1. Note that the algorithm may offer no recommendations when constraint (1c) is not binding. Problem (1) is a linear program, whose optimal objective value is denoted by $R(\boldsymbol{k}, \xi)$.

B. Two-Stage Stochastic Programming Problem

Recall that our BSS-Net consists of a single CF and M BSSs. The per-battery charging and delivery cost from the CF to BSS m is assumed to be α and β_m, respectively. Meanwhile, we also assume that the maximum (minimum) number of FBs that can be delivered is denoted by $\overline{k}$ ($\underline{k}$). We then formulate the long-term initial inventory planning problem as a two-stage problem as follows:

$$\max_{k_m}\ \mathbb{E}[R(\boldsymbol{k}, \xi)] - \alpha \sum_{m\in\mathcal{M}} k_m - \sum_{m\in\mathcal{M}} \beta_m k_m \tag{2a}$$

$$\text{s.t. } \underline{k} \le \sum_{m\in\mathcal{M}} k_m \le \overline{k} \tag{2b}$$

$$k_m \ge 0 \quad \forall m \in \mathcal{M} \tag{2c}$$

where $R(\boldsymbol{k}, \xi)$ is the optimal objective value of Problem (1).

Note that Problems (1) and (2) contribute to a two-stage stochastic programming problem, which is typically computationally cumbersome [26]. In particular, in the first stage, the decision variable $\boldsymbol{k}$ has to be chosen before a particular realization ξ is observed, whereas in the second stage, the decision $x_{n,m}^j(S)$ is made after the observation of the realization ξ and depends on the first-stage decision $\boldsymbol{k}$. In the following, we refer to Problem (2) as the first stage and Problem (1) as the second stage. To efficiently solve this two-stage stochastic programming problem, we propose the following Proposition 1 regarding the concavity of $R(\boldsymbol{k}, \xi)$. Here, $\boldsymbol{k}$ is relaxed as a continuous variable.

Proposition 1: $R(\boldsymbol{k}, \xi)$ is concave over $\boldsymbol{k} \in \mathbb{K}$, where $\mathbb{K}$ is the feasible set of the initial inventory in Problem (2).

The proof of Proposition 1 is explained in [27, Appendix A], due to the space limit.

C. Sample Average Approximation

In Section III-A, the uncertainties of EV arrivals are modeled as a random variable ξ. In reality, we possibly have an infinite number of possible realizations; thus, it is difficult to know the exact probability (distribution) of ξ. To deal with this difficulty, a common approach is to reduce the scenario set to a manageable size by using the Monte Carlo simulation [28]. Specifically, we can randomly generate a sample set $\boldsymbol{\xi} \triangleq \{\xi_1, \ldots, \xi_{\bar{\ell}}\}$, where each sample $\xi_\ell \in \boldsymbol{\xi}$ is assumed to be independent and identically distributed. The expectation term $\mathbb{E}[R(\boldsymbol{k}, \xi)]$ in the first stage (2a) can then be approximated by the sample average approximation (SAA) method, $\mathbb{E}[R(\boldsymbol{k}, \xi)] = 1/\bar{\ell} \sum_{\ell=1}^{\bar{\ell}} R(\boldsymbol{k}, \xi_\ell)$. Here, the law of large number can be applied when $\bar{\ell}$ is sufficiently large. In our problem, the sample set $\boldsymbol{\xi}$ can be derived from the historical data. Based on Proposition 1 and the aforementioned SAA technique, we can transform the two-stage stochastic programming problem in (1) and (2) into a solvable single-stage problem as follows:

$$\begin{aligned}\max_{\boldsymbol{k}, x_n^\ell(S)}\ & \frac{1}{\bar{\ell}} \sum_{\ell=1}^{\bar{\ell}} \sum_{n\in\mathcal{N}} \sum_{S\subseteq\mathcal{A}} x_n^\ell(S) \sum_{(m,j)\in S} r_m^j p_{n,m}^j(S) \\ & - \alpha \sum_{m=1}^{M} k_m - \sum_{m=1}^{M} \beta_m k_m \\ \text{s.t. } & (1\text{b}) - (1\text{d}),\ (2\text{b}) - (2\text{c})\end{aligned} \tag{3}$$

where $x_n^\ell(S)$ is the recommendation decision for the sample ξ_ℓ. We can observe that Problem (3) is a linear program, which can be efficiently solved by commercial solvers even in a practical large-scale BSS-Net. Based on the optimal inventory solution $\boldsymbol{k}^*$, we next present the real-time V2S recommendation.

IV. Online V2S Recommendation

In this section, we propose a randomized online V2S recommendation algorithm based on the value of obtained optimal initial inventory and rigorously prove its worst case performance guarantee.

A. Statement of the Online Algorithm

The basic idea behind our online V2S recommendation coincides with the relationship between supply and demand in economics. When the supply level is high (adequate FBs), the operator tends to use low selling prices to serve the FB demand. Once the supply level decreases, the operator becomes more conservative to serve demand by high prices. In that case, the operator can reserve some FBs for possible future customers that can accept higher prices to make more profits. Other important factors (such as traffic conditions) that impact the V2S routing decisions are captured by customers' choice probability. Based on customers' choice probability, we estimate the expected revenue for all possible recommendations and provide an appropriate recommendation to maximize the expected revenue.

Therefore, we set a threshold price for using one unit of remaining FBs, which is commonly called bid price. Intuitively, when the remaining inventory decreases, the potential value of one unit of FBs increases. In [13], a piecewise increasing value function ϕ_m is designed based on the relaxation of continuous inventory levels, which is the bid price function in an asymptotic case. Hereinafter, we will use a value function to indicate the bid price function. Specifically, with the assumption of infinite initial inventories, the asymptotic value function ϕ_m is defined on a continuous fraction of remaining inventory for each BSS $m \in \mathcal{M}$ and segmented

Algorithm 1 Two-Step Online Recommendation Algorithm

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1: Inventory Planning: Compute optimal inventory k*_m by solving (3). Set k_m ← k*_m, V_{0,m} ← 0, m ∈ M.
2: Randomized: {L̃^j_m}_{j∈J}, φ̃_m, ∀m ∈ M,
3: for n = 1, 2, ..., N do
4:    Compute S* by solving (4),
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$$\max_{S\in\mathcal{A}} \sum_{(m,j)\in S} p^j_{n,m}(S)\left(\tilde{\phi}(\tilde{L}^j_m) - \tilde{\phi}(\frac{V_{n-1,m}}{k_m})\right), \quad (4)$$

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5:    if the optimal value of (4) is strictly positive then
6:       Recommend S* to EV n.
7:       if customer n accepts any option (m*_n, j*_n) ∈ S* then
8:          R_on ← R_on + r^{j*_n}_{m*_n}.
9:          V_{n,m*_n} ← V_{n-1,m*_n} + 1.
10:      end if
11:   else
12:      Offer no recommendation.
13:   end if
14: end for
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by a group of segment borders $\{L^j_m\}_{j\in\mathcal{J}_m}$. However, in that case, the true value of the initial inventory is neglected with only asymptotic theoretical results derived, which is explained in detail in Section IV.

In comparison, we consider a more general case for finite initial inventory, which is consistently set as the optimal solution of (3). Based on the aforementioned design logic, we design a discrete piecewise increasing value function $\tilde{\phi}_m, m \in \mathcal{M}$ over a discrete fraction of remaining inventory, namely, a multiple of $1/k_m$. Furthermore, the discrete value function $\tilde{\phi}_m$ is segmented by a group of segment borders $\{\tilde{L}^j_m\}_{j\in\mathcal{J}_m}$, which only take value as multiples of $1/k_m$, such that $0 = \tilde{L}^0_m \le \cdots \le \tilde{L}^J_m = 1$ and $0 = \tilde{\phi}_m(\tilde{L}^0_m) \le \cdots \le \tilde{\phi}_m(\tilde{L}^J_m)$. Here, a strong intuition is to design $\tilde{\phi}_m$ and $\tilde{L}^j_m$ based on the asymptotic results $\phi_m, L^j_m, m \in \mathcal{M}$. However, the fact that asymptotic segment border L^j_m has no need to be a multiple of $1/k_m$ hinders us from directly using L^j_m as the segment border. Therefore, a randomized procedure is needed to handle the rounding error from L^j_m to $\tilde{L}^j_m$, which will be explained in detail in Section IV-C. Interested readers can also refer to [13] for more about the derivation of asymptotic ϕ_m and L^j_m. We next present the details of a two-step online recommendation algorithm in Algorithm 1 (ALG-1).

In Algorithm 1, the initial inventory is set as the optimal solution $k^*_m, m \in \mathcal{M}$ of (3), and $V_{0,m}$ is initialized to be 0 for all BSSs. The detailed randomized procedure mentioned in Line 2 is present in Section IV-D. In (4), the pseudorevenue is measured based on the value on segment border $\tilde{\phi}_m(\tilde{L}^j_m)$ and value of per-unit remaining FBs $\tilde{\phi}_m(V_{n-1,m}/k_m)$ before serving customer n. The expected pseudorevenue is maximized in order to offer recommendation S^*. In the following, we show that the performance of ALG-1 is theoretically guaranteed when the value function $\tilde{\phi}_m, \forall m \in \mathcal{M}$, is wisely designed.

B. Performance Metric and Dual Formulation

The value function $\tilde{\phi}_m, \forall m \in \mathcal{M}$, is designed through a principled online primal–dual analysis (see [29], [30]). Before the analysis, we introduce the definition of competitive ratio, which works as the performance metric of the online V2S recommendation algorithm to quantify the worst case performance. In the following, we give a formal definition of the competitive ratio.

Definition 1: An online V2S recommendation algorithm is $\tilde{c}$-competitive if $\mathsf{R_{on}} \ge \tilde{c} \cdot \mathsf{OPT}$ holds for all possible future EV arrivals, where OPT is the maximal total revenue obtained by knowing all the future information.

Note that $\tilde{c} \in [0, 1]$ is a constant, and $\tilde{c}$'s getting closer to 1 means the online algorithm achieving better performance. Utilizing the competitive ratio as our online algorithm performance metric, $\{\tilde{\phi}_m\}_{\forall m\in\mathcal{M}}$ can be derived through the online primal–dual analysis of Problem (1), which works as the primal problem. The dual problem of (1) is stated as follows:

$$\min_{\lambda_m,\mu_n} \sum_{m\in\mathcal{M}} k_m\lambda_m + \sum_{n\in\mathcal{N}} \mu_n \quad (5a)$$

$$\text{s.t.} \sum_{(m,j)\in S} \lambda_m p^j_{n,m}(S) + \mu_n \ge \sum_{(m,j)\in S} r^j_n p^j_{n,m}(S)$$
$$\forall n \in \mathcal{N} \quad \forall S \subseteq \mathcal{A} \quad (5b)$$
$$\lambda_m, \mu_n \ge 0 \quad \forall m \in \mathcal{M}\ \forall n \in \mathcal{N} \quad (5c)$$

where $\lambda_m, m \in \mathcal{M}$ and $\mu_n, n \in \mathcal{N}$ are the dual variables related to (1b) and (1c), respectively. Based on the primal problem (1) and the dual problem (5), we have the following Lemma 1 and defer its proof in [27, Appendix B].

Lemma 1: If there exists a constant $0 < \tilde{c} \le 1$ such that the following increment inequality holds:

$$P_n - P_{n-1} \ge \tilde{c} \cdot (D_n - D_{n-1}) \quad \forall n \in \mathcal{N} \quad (6)$$

then the online algorithm is $\tilde{c}$-competitive, where P_n and D_n are, respectively, the primal and dual objective values of the online algorithm after serving the nth customer.

C. Online Primal-Dual Analysis

Based on Lemma 1, we analyze the design principle of Algorithm 1 and propose Theorem 1.

Theorem 1: If the value function $\tilde{\phi}_m, \forall m \in \mathcal{M}, n \in \mathcal{N}$, satisfies (7) and (8), then ALG-1 is $\tilde{c}$-competitive

$$k_m\left(\tilde{\phi}_m\left(\frac{V_{n-1,m}+1}{k_m}\right) - \tilde{\phi}_m\left(\frac{V_{n-1,m}}{k_m}\right)\right) + \tilde{\phi}_m(\tilde{L}^j_m) - \tilde{\phi}_m\left(\frac{V_{n-1,m}}{k_m}\right) \le \frac{r^j_m}{\tilde{c}} \quad (7)$$

$$\mathbb{E}\left[\tilde{\phi}^j_m(\tilde{L}^j_m)\right] \ge r^j_m. \quad (8)$$

The proof for Theorem 1 is elaborated through standard online primal–dual analysis, which follows a similar logic in [13] until the derivation of a sufficient condition to guarantee the increment inequality shown in Lemma 1. Due to space limit, we defer the detailed proof of Theorem 1 in [27,

Appendix C] and only introduce the main logic here to give some intuition. Different from [13], ALG-1 generates dual variables $\lambda_m = \mathbb{E}[\tilde{\phi}_m(V_{N,m}/k_m)]$ and $\mu_n = \mathbb{E}[U_n]$, where U_n can be interpreted as the pseudorevenue earned by serving EV n. If EV n chooses one recommendation (m, j), then $U_n = \tilde{\phi}_m(\tilde{L}_m^j) - \tilde{\phi}_m(V_{n-1,m}/k_m)$, $\forall(m, j) \in S^*$; otherwise, $U_n = 0$. Thus, the pseudorevenue is ensured exactly zero by recommending (m, j) when the consumed fraction of initial inventory reaches $\tilde{L}_m^j$. To prove Theorem 1, we first elaborate that dual variables generated by ALG-1 are feasible when (8) is satisfied. Next, we complete the proof by showing that the primal and dual objectives P_n and D_n achieve the increment inequality (6), if (7) holds.

D. Value Function and Competitive Ratio

Now, we only need to focus on how to design the value function $\tilde{\phi}_m, m \in \mathcal{M}$ to ensure the validity of (7) and (8).

1) Asymptotic: The design of $\tilde{\phi}_m$ based on Theorem 1 for a finite value of $k_m, m \in \mathcal{M}$ is non-trivial, as it is very challenging to directly obtain the segment border $\tilde{L}_m^j$. However, when k_m approaches infinity, we can asymptotically derive a group of continuous segment border $L_m^j, m \in \mathcal{M}$ and a continuous asymptotic value function ϕ_m. Therefore, we first introduce an asymptotic case when $k_m \to \infty$ [13], such that $(V_{n-1,m}/k_m)$ can be represented by a continuous variable w_m. The dual variable U_n is set as $r_m^j - \phi_m(V_{n-1,m}/k_m)$ since $\phi_m(L_m^j) = r_m^j$. We then rewrite (7) as the following differential equation:

$$\phi_m'(w_m) - \phi_m(w_m) \le r_m^j\left(\frac{1}{c} - 1\right). \tag{9}$$

We solve (9) on each segment $w_m \in [L_m^{j-1}, L_m^j)$ by binding the inequality. With initial conditions $\phi_m(L_m^{j-1}) = r_m^{j-1}$ and $\phi_m(L_m^j) = r_m^j$, we obtain an asymptotic value function $\phi_m, m \in \mathcal{M}$ as

$$\phi_m(w_m) = \frac{r_m^j - r_m^{j-1}}{e^{L_m^j - L_m^{j-1}} - 1}\left(e^{w_m - L_m^{j-1}} - 1\right) + r_m^{j-1}, \tag{10}$$

where ϕ_m is an asymptotic function to the true value function $\tilde{\phi}_m$ when the initial inventory k_m is extremely large. c and $\{L_m^j\}_{j\in\mathcal{J}_m}$ are derived by setting the value of (11) equal for all $j \in \mathcal{J}_m$ such that c can be maximized

$$1 - e^{-L_m^1} = \frac{1 - e^{L_m^{j-1} - L_m^j}}{1 - r_m^{j-1}/r_m^j} \tag{11}$$

$$c = \min_m 1 - e^{-L_m^1} \tag{12}$$

where the value of c is irrelevant to $k_m, m \in \mathcal{M}$.

2) Randomized: Based on the results in asymptotic case, we investigate a more general case, where $k_m, m \in \mathcal{M}$ takes finite values. Specifically, we design a discrete value function $\tilde{\phi}_m, m \in \mathcal{M}$, which only takes value from a discrete set $q \in \{0, 1/k_m, 2/k_m, \ldots, 1\}$. Inspired by the design of $\phi_m(w_m)$, we divide the range of q into different segments $[\tilde{L}_m^{j-1}, \tilde{L}_m^j), \forall j \in \mathcal{J}_m$. The new segment border $\tilde{L}_m^j$ can be calculated by rounding L_m^j to a value of a multiple of $1/k_m$. However, the random rounding error from $\tilde{L}_m^j$ to L_m^j greatly increases the difficulty in deriving a closed-form competitive ratio $\tilde{c}$. Thus, we need to properly design a randomized procedure to locate $\{\tilde{L}_m^j\}_{j\in\mathcal{J}}$ and obtain the corresponding value function $\tilde{\phi}_m(q)$ such that ALG-1 can achieve a competitive ratio $\tilde{c}$, which is related to the value of $k_m, m \in \mathcal{M}$.

Inspired by [25], we start a randomized procedure by using a random seed γ uniformly drawn from $[0, 1]$ to capture the randomness in rounding error from $\tilde{L}_m^j$ to L_m^j and proceed to design $\tilde{\phi}_m$ based on $\phi_m, m \in \mathcal{M}$. The randomized procedure is stated as follows.

1) For $\forall j \in \mathcal{J}_m, m \in \mathcal{M}$, $\tilde{L}_m^j = \frac{\lfloor L_m^j k_m \rfloor + 1}{k_m}$, if $\gamma < L_m^j k_m - \lfloor L_m^j k_m \rfloor$. Otherwise, $\tilde{L}_m^j = ((\lfloor L_m^j k_m \rfloor)/k_m)$.
2) For $q \in [\tilde{L}_m^{j-1}, \tilde{L}_m^j)$, the randomized value function is then set as

$$\tilde{\phi}_m(q) = \sum_{g=1}^{j-1}\left(\Delta_m^g\right)\frac{e^{\tilde{h}_m^g} - 1}{e^{h_m^g} - 1} + \left(\Delta_m^j\right)\frac{e^{q - \tilde{L}_m^{j-1}} - 1}{e^{h_m^j} - 1} \tag{13}$$

where $h_m^j = L_m^j - L_m^{j-1}, \tilde{h}_m^j = \tilde{L}_m^j - \tilde{L}_m^{j-1}$, and $\Delta_m^j = r_m^j - r_m^{j-1}, j \in \mathcal{J}_m$ for notational convenience. By randomly rounding error between $\tilde{L}_m^j$ and L_m^j, the value $\tilde{\phi}(\tilde{L}_m^j)$ has a cumulative deviation from $\phi_m(L_m^j)$. We capture this deviation from random rounding by the first term of (13). If we replace $\{\tilde{L}_m^j\}_{j\in\mathcal{J}_m}$ by $\{L_m^j\}_{j\in\mathcal{J}_m}$, (13) immediately recovers (10).

Next, we move on to verify that procedures 1) and 2) satisfy (7) and (8) in Theorem 1 by the following two steps.

1) We prove the satisfaction of (8) inductively. First, we can observe that $\tilde{\phi}_m(\tilde{L}_m^0) = r_m^0 = 0$. By assuming (8) that holds for $j - 1$ and substituting $q = \tilde{L}_m^j$ and $q = \tilde{L}_m^{j-1}$ into (13), we can derive $\tilde{\phi}_m(\tilde{L}_m^j) = \tilde{\phi}_m(\tilde{L}_m^{j-1}) + (\Delta_m^j)((e^{\tilde{h}_m^j} - 1)/(e^{h_m^j} - 1))$. Thus, we have

$$\mathbb{E}\left[\tilde{\phi}\left(\tilde{L}_m^j\right)\right] \ge r_m^{j-1} + \left(r_m^j - r_m^{j-1}\right)\frac{e^{\mathbb{E}\left[\tilde{h}_m^j\right]} - 1}{e^{h_m^j} - 1} = r_m^j \tag{14}$$

where the equality holds for $\mathbb{E}[\tilde{L}_m^j] = ((\lfloor L_m^j k_m \rfloor + 1)/k_m)(L_m^j k_m - \lfloor L_m^j k_m \rfloor) + ((\lfloor L_m^j k_m \rfloor)/k_m)(1 - L_m^j k_m + \lfloor L_m^j k_m \rfloor) = L_m^j$ such that $\mathbb{E}[\tilde{h}_m^j] = h_m^j$. Therefore, we prove the satisfaction of (8).
2) Next, we validate (7) by substituting (13) and $q = (V_{n-1,m}/k_m)$ into the left-hand side of (7). For $q \in [\tilde{L}_m^{j-1}, \tilde{L}_m^j)$, we obtain

$$\begin{aligned} &k_m\left(\tilde{\phi}_m\left(q + \frac{1}{k_m}\right) - \tilde{\phi}_m(q)\right) + \tilde{\phi}_m\left(\tilde{L}_m^j\right) - \tilde{\phi}_m(q) \\ &= \frac{\Delta_m^j}{e^{h_m^j} - 1}\left((k_m - (1 + k_m)e^{-1/k_m})e^{q + \frac{1}{k_m} - \tilde{L}_m^{j-1}} + e^{\tilde{h}_m^j} - 1\right) \\ &\le \frac{\Delta_m^j}{e^{h_m^j} - 1}\left(\left(k_m - (1 + k_m)e^{-\frac{1}{k_m}}\right)e^{\tilde{h}_m^j} + e^{\tilde{h}_m^j}\right) \qquad (15) \\ &\le \frac{\Delta_m^j}{1 - e^{-h_m^j}}e^{\frac{1}{k_m}}(1 + k_m)\left(1 - e^{\frac{-1}{k_m}}\right) \qquad (16) \end{aligned}$$

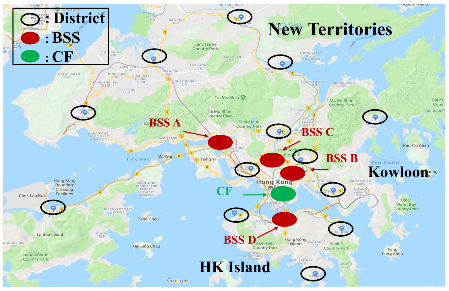


Fig. 2. Illustration of a Hong Kong map with 18 districts.

$$= r_m^j \frac{(1+k_m)\left(e^{\frac{1}{k_m}}-1\right)}{\left(1-e^{-L_m^1}\right)} \tag{17}$$

where the inequality (15) holds for $k_m - (1 + k_m)e^{-1/k_m} \geq 0$ and $e^{q+(1/k_m)} \leq e^{\tilde{L}_m^j}$. Inequality (16) holds for the fact $|\tilde{h}_m^j - h_m^j| \leq (1/k_m)$, which can be easily obtained from 1). The equality (17) is derived by (11). Therefore, we can validate (7) and derive the competitive ratio $\tilde{c}$ according to (7) and (17) as

$$\tilde{c} = \min_{m \in \mathcal{M}} \frac{\left(1-e^{-L_m^1}\right)}{(1+k_m)\left(e^{\frac{1}{k_m}}-1\right)}. \tag{18}$$

Note that $\lim_{k_m \to \infty}(1 + k_m)(e^{(1/k_m)} - 1) = 1$, and we can immediately recover c from $\tilde{c}$ when $k_m \to \infty$.

V. Simulation Results

In this section, we evaluate the performance of our proposed two-step scheme based on a case study of Hong Kong (HK), by showing the empirical performance of our scheme in different cases and comparing it to other benchmarks.

A. Setup of the Case Study

1) Setup of the BSS-Net Model: We consider a BSS-Net based on a simplified transportation network of HK, including a total of 18 districts, 32 main roads, four BSSs, and one centralized CF, as shown in Fig. 2. We provide battery-swapping services for an electric taxi company covering all three areas (New Territories, Kowloon, and HK Island) of HK, where all batteries are uniform with a capacity $\mathcal{C} = 80$ kWh. The battery-swapping prices in each BSS are computed based on the battery capacity and the electricity price in each area of HK (1.119 HKD/kWh for HK Island and 0.884 HKD/kWh for New Territories and Kowloon), which are shown in Table I. Specifically, the price is defined as the product of the electricity cost of charging the battery and a profit factor. In this article, we use 1.4 and 1.8 as the profit factors of low and high prices in each BSS.

2) Setup of the Choice Model: We set up the choice model of EV customers in our case study based on the multinomial logit model [22]–[24]. First, we model the preference of EV n to combination (m, j) as

$$\tilde{p}_{n,m}^j = \theta_n^0 + \theta_n^1 / d_{n,m} + \theta_n^2 / (r_{m,j})^2 \tag{19}$$

TABLE I

Settings of Battery Prices (HKD) in the Four-BSS System

	BSS A	BSS B	BSS C	BSS D
Area	New Territories	Kowloon	Kowloon	HK Island
Low Price	99	99	99	125
High Price	128	128	128	161

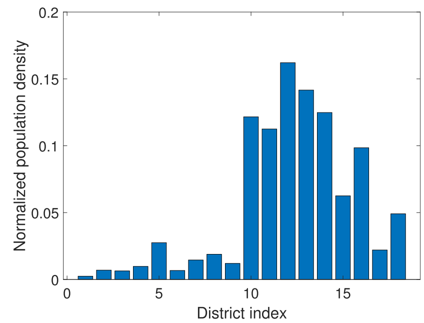


Fig. 3. Population density of 18 districts.

which indicates an inversely proportional relationship to driving distance $d_{n,m}$ and price r_m^j. The parameters $(\theta_n^0, \theta_n^1, \theta_n^2)$ are uniformly drawn from $[0, 1], [5, 20], [10\,000, 20\,000]$. Given a recommendation S, the probability for EV n to choose (m, j) from S is estimated as

$$p_{n,m}^j(S) = \frac{e^{\tilde{p}_{n,m}^j}}{\sum_{(m,j)\in S} e^{\tilde{p}_{n,m}^j} + e^{\tilde{p}_{n,0}}} \tag{20}$$

where $\tilde{p}_{n,0}$ is the preference for EV n to accept no recommendation, derived by setting $\theta_n^1 = \theta_n^2 = 0$.

3) Setup of EVs' Arrival Instance: All the following numerical tests are implemented on EVs' arrival instances $\xi \triangleq \{\mathcal{N}, \{p_{n,m}^j(S)\}_{S,n,(m,j)}\}$. We estimate the average total number of EVs by a nominal value 2000 multiplied by a loading factor (LF) ρ, which can be interpreted as the average number of EV customers for one unit of initial FBs, such that the realized total number N is a Poisson random variable with mean 2000ρ. The probability of an EV appearing in a particular district y, denote by $\mathbb{P}_n^y$, is estimated according to the normalized population densities of the 18 districts of HK which are visualized in Fig. 3. The probability of an EV appearing at the streets $e \in \mathcal{E}(y)$ connected to the district y is calculated as $\mathbb{P}_n^{y,e} = (G_e/(\sum_{e\in\mathcal{E}(y)} G_e))\mathbb{P}_n^y$, where G_e is the street length observed from Google map data. Also, we estimate the exact location of the EV on street e by an uniform distribution $\mathcal{U}(g), \forall g \in [0, G_e]$. Therefore, we can generate the driving distance $d_{n,m}, m \in \mathcal{M}$. Based on (20), we can obtain a complete EVs' arrival instance ξ.

B. Optimal Initial Inventory

To determine the optimal initial inventory in the first step of our two-step scheme, we apply SSA by generating a large number of EVs' arrival instances. Specifically, a total of $\bar{\ell} = 100$ instances are considered in (3), for any customer size N with $\rho \in \{0.75, 0.8, \ldots, 1.2\}$, to obtain the corresponding optimal initial inventories, which are shown in Fig. 4. If the

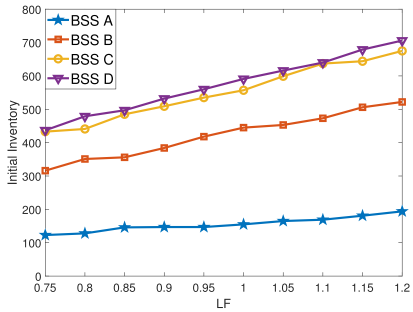
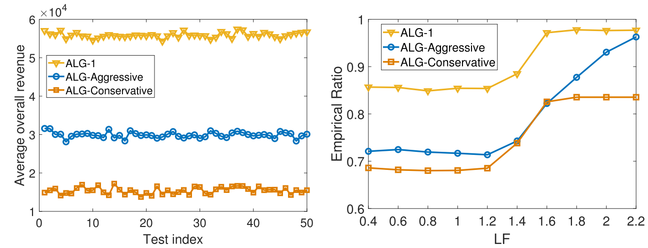
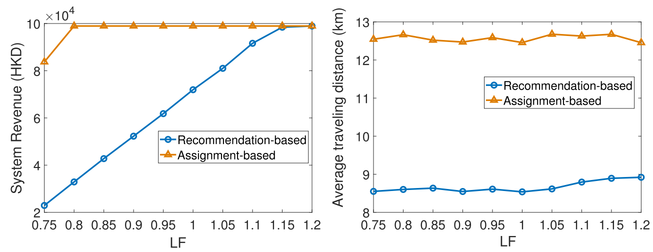


Fig. 4. Initial inventory for different LFs.



(a) Average overall revenue (b) Empirical ratio

Fig. 5. Performance comparison between online algorithms. (a) Average overall revenue. (b) Empirical ratio.



(a) Overall revenue (b) Traveling distance

Fig. 6. Performance comparison with assignment-based V2S routing. (a) Overall revenue. (b) Traveling distance.

BSS-Net operator has a good prediction of the customer size for the next service horizon, then the corresponding results can be used to build up the initial inventory. Note that the uncertainty of EV customer size in the online algorithm is captured by tuning LF.

C. Performance Comparison With Other Benchmarks

1) Revenue Comparison: We set the initial inventory as the optimal solution for $N = 2000$ obtained in Section V-B, which are $(155, 445, 557, 591)$. We compare the average overall revenue between Algorithm 1 and two benchmark online algorithms: ALG-Conservative and ALG-Aggressive, which always recommend feasible BSSs with low prices and high prices without considering the inventory level, respectively. Specifically, we calculate the average revenue of different LFs for 50 times. Within each time, we generate ten instances for LFs $\rho \in \{0.75, 0.8, \ldots, 1.2\}$ and then calculate the corresponding average. We present the revenue results in Fig. 5(a), where ALG-1 significantly outperforms other benchmarks.

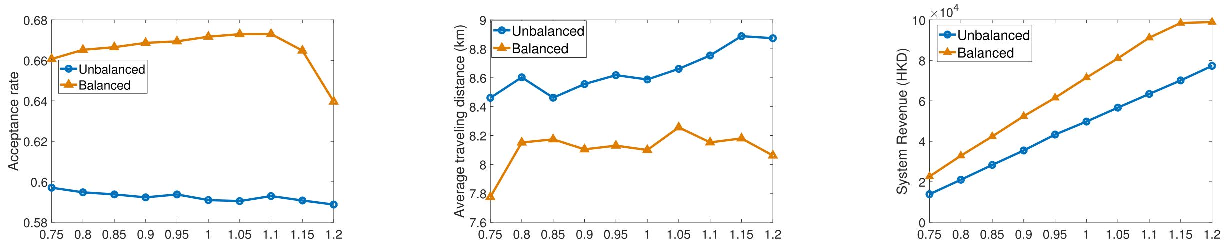
2) Empirical Ratio Comparison: We further measure the performance of three algorithms by an empirical ratio $\mathsf{R}_{\mathsf{on}}/\mathsf{OPT}$, where R_{on} is the total revenue for an online algorithm in one instance and OPT represents the corresponding revenue upper bound obtained from (3). We compare the average empirical ratio of ten instances for $\rho \in \{0.75, 0.8 \ldots, 1.2\}$ in Fig. 5(b). From the testing results, ALG-1 achieves a greater empirical ratio for a wide range of LFs. Note that for ALG-Aggressive, the empirical ratio gets close to 1 when LF becomes large enough. This is reasonable since ALG-Aggressive can serve all EVs with high prices when the customer size is very large. Furthermore, we can notice that the empirical ratio is much greater than the theoretical result shown in (18), which indicates that our online algorithm works well in practice.

D. Performance Comparison With Assignment-Based V2S Routing

In this section, we compare the system revenue and average traveling distance between our proposed recommendation-based V2S routing method and the assignment-based V2S routing aforementioned in Section I. The assignment-based routing can be regarded as a special case of recommendation-based routing, where the probability of accepting the assigned (station, price) is 1. With the same settings of customer size and initial inventory in Section V-C, we first show the overall system revenues of the assignment- and recommendation-based methods in Fig. 6(a). We can observe that the revenue of the assignment-based method is greater since customers always accept the (station, price) assignments and more FBs are consumed than the recommendation-based method. However, when the customer size becomes larger, more recommendations are accepted in the recommendation-based method, and the overall revenue gets closer to the assignment-based one. Fig. 6(b) shows the average traveling distances for the two methods. We can see that the distance in the assignment-based method is much larger since customers have to accept the assignment even if the distance is long.

E. Performance of the Two-Step Scheme for Different Demand Patterns

In this section, we compare the performance of our two-step scheme for different battery-swapping demand patterns. We consider the scenario of unbalanced demand when there is a big event in the stadium in HK Island. In this case, EVs (electric taxis) concentrate on HK Island. Thus, the FB stock in BSS D is rapidly consumed. In addition, the EVs in HK Island are unwilling to pass through the harbor tunnel between HK Island and Kowloon to get FBs in BSSs A–C and may choose other refueling methods (e.g., plug-in charging). For the scenario of balanced demand, EVs appear in the three areas of HK based on local population densities, as shown in Fig. 3. With the same settings of customer size and initial inventory in Section V-C, we first show the average recommendation acceptance rate of EVs in Fig. 7(a). We can observe that the EVs in the case of unbalanced demand have a lower rate of accepting the recommendations from the operator since most

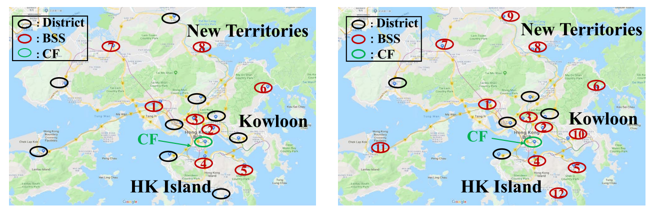


(a) Acceptance rate (b) Traveling distance (c) Overall revenue

Fig. 7. Performance comparison for different demand patterns. (a) Acceptance rate. (b) Traveling distance. (c) Overall revenue.

TABLE II
AVERAGE COMPUTATION TIMES FOR INITIAL INVENTORY PLANNING AND V2S RECOMMENDATION

Test systems	Initial inventory planning	V2S Recommendation
4-BSS	1527.9 sec	0.00004 sec
8-BSS	5137.4 sec	0.0013 sec
12-BSS	44174 sec	0.0074 sec



(a) The 8-BSS system (b) The 12-BSS system

Fig. 8. (a) 8-BSS and (b) 12-BSS systems based on an HK map.

of the EVs either choose BSS D or reject the recommendation. Furthermore, the average traveling distance in the case of unbalanced demand is greater than that in the case of balanced demand because some EVs probabilistically choose to travel long distances across areas to get FBs in BSSs A–C. The overall system revenue under balanced demand is significantly better than that under the unbalanced demand based on the fact that more FBs are consumed in the BSS-Net. The results of average traveling distance and overall revenue are shown in Fig. 7(b) and (c), respectively.

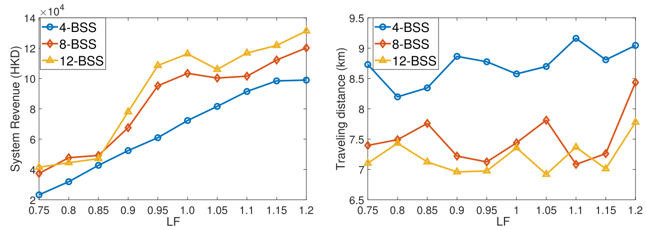
F. Scalability Illustration

In order to show the scalability of our method for larger systems, we have performed additional simulations on 8-BSS and 12-BSS systems, as shown in Fig. 8. With the same settings of customer size and LFs in Section V-C, we show the computation times for the long-term initial inventory and short-time V2S recommendation in Table II and the overall system revenue and average traveling distance for an EV customer in the 4-BSS, 8-BSS, and 12-BSS systems in Fig. 9.

From Table II, we can observe that the decision of real-time recommendation for a customer can be made very fast in different scales of systems. The computation time of the initial inventory planning is several hours for the 12-BSS system, which is acceptable since the decision on the initial inventory is made in an offline manner (e.g., day-ahead or week-ahead). Fig. 9(a) shows that the overall system revenues in the 8-BSS and 12-BSS systems are greater than that of the 4-BSS system since the customers have more choices and are more willing to accept the recommendation. However, the revenues in the 8-BSS and 12-BSS systems are close due to the total number of districts in HK that is limited to 18. Fig. 9(b) shows that the average traveling distances in the 8-BSS and 12-BSS systems are smaller than that in the 4-BSS system, as EVs have more choices to choose nearby BSSs. Similarly, the traveling distance in the 12-BSS system is slightly smaller than that of the 8-BSS system.

G. Discussions on the Operator's Response Delay

In this section, we discuss the scenario when the total number of requests becomes very large and multiple requests arrive in a very short time. In practice, the operator will ask the customers to respond to the recommendations within a time window and consider no response as a rejection decision. Since it may take quite a long time to receive all the responses from all previous requests, the operator needs to set an appropriate deadline τ (e.g., a few seconds or even zero) to respond to each request. In this way, the operator will make recommendations to each request within τ. When the total number of requests is small, the responses from all previous customers can be received within τ with high probabilities and our algorithm can ensure the competitive ratio theoretically. When the total number of requests is large, although the operator may not receive the responses from all customers, the operator can still make recommendations based on the current system state. Since in this case, the initial inventory is relatively large (determined in the long-term initial inventory planning problem), the system state does not change a lot before and after the unreceived responses are considered. Thus, the resulting recommendation decision can be a reasonably good one. To show this, we numerically compare the performances of two algorithm implementations. In the first implementation, the operator makes recommendations only after receiving all the responses. This design has a theoretical performance guarantee but needs a long time to make recommendations for each request. In the second implementation, the operator makes recommendations to each request within a deadline $\tau = 0$. We compute the system revenues in both cases for a total of 2000δ requests, $\delta \in \{1, 2, \ldots, 10\}$, where a batch of



(a) System revenue (b) Traveling distance

Fig. 9. Comparison for different scale systems. (a) System revenue. (b) Traveling distance.

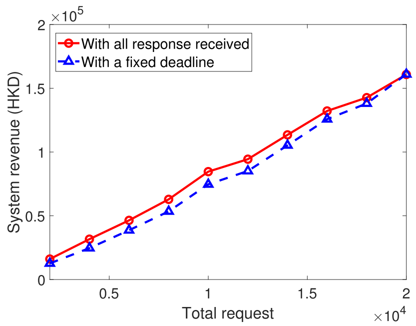


Fig. 10. Comparison of revenues in the cases with and without response delay.

50δ requests arrive at the same time and a total of 10 batches are considered in one day. The comparison results of revenues are shown in Fig. 10. We can observe that the revenues in the two cases are very close. The average revenue decrease in the case with a deadline $\tau = 0$ compared to that in the case with all responses received is only 5.39%.

VI. Conclusion

In this article, we investigated the two-timescale decision-making problem of a BSS-Net in an urban area and designed a two-step scheme to jointly optimize the initial inventory planning of BSSs and the real-time V2S routing of EVs. In particular, we first determined the long-term initial inventory in each BSS by optimally solving a two-stage stochastic programming problem. Based on the obtained optimal initial inventory, we proposed a real-time V2S routing strategy through a two-step online algorithm. Compared with existing work in an asymptotic case, we investigate a more general case considering the discrete nature of the value function. Specifically, we designed a randomized procedure and an associated online algorithm, where the worst case performance of the proposed algorithm is proved to be theoretically guaranteed by a closed-form competitive ratio. Furthermore, we conducted a case study in a simplified HK transportation network, and the numerical results demonstrated that we can obtain the initial inventory for different customer sizes so as to maximize the expected revenue of the BSS-Net. Meanwhile, it is also validated by our simulations that the proposed two-scheme is scalable and achieves better performance compared with other benchmarks.

References

[1] E. Ungar and K. Fell, "Plug in, turn on, and load up," *IEEE Power Energy Mag.*, vol. 8, no. 3, pp. 30–35, May 2010.

[2] Z. Jian-hua, "BYD EV charging challenges solutions," in *Proc. 4th Int. Conf. Power Electron. Syst. Appl.*, Jun. 2011, pp. 1–4.

[3] B. Avci, K. Girotra, and S. Netessine, "Electric vehicles with a battery switching station: Adoption and environmental impact," *Manage. Sci.*, vol. 61, no. 4, pp. 772–794, Apr. 2015.

[4] H.-Y. Mak, Y. Rong, and Z.-J.-M. Shen, "Infrastructure planning for electric vehicles with battery swapping," *Manage. Sci.*, vol. 59, no. 7, pp. 1557–1575, Jul. 2013.

[5] X. Tan, G. Qu, B. Sun, N. Li, and D. H. K. Tsang, "Optimal scheduling of battery charging station serving electric vehicles based on battery swapping," *IEEE Trans. Smart Grid*, vol. 10, no. 2, pp. 1372–1384, Mar. 2019.

[6] Q. Kang, J. Wang, M. Zhou, and A. C. Ammari, "Centralized charging strategy and scheduling algorithm for electric vehicles under a battery swapping scenario," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 3, pp. 659–669, Mar. 2016.

[7] Q. Dai, T. Cai, S. Duan, and F. Zhao, "Stochastic modeling and forecasting of load demand for electric bus battery-swap station," *IEEE Trans. Power Del.*, vol. 29, no. 4, pp. 1909–1917, Aug. 2014.

[8] R. S. Widrick, S. G. Nurre, and M. J. Robbins, "Optimal policies for the management of an electric vehicle battery swap station," *Transp. Sci.*, vol. 52, no. 1, pp. 59–79, Jan. 2018.

[9] P. You *et al.*, "Scheduling of EV battery swapping—Part I: Centralized solution," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 4, pp. 1887–1897, Dec. 2018.

[10] P. You *et al.*, "Scheduling of EV battery swapping—Part II: Distributed solutions," *IEEE Trans. Control Netw. Syst.*, vol. 5, no. 4, pp. 1920–1930, Dec. 2018.

[11] M. P. Fanti, A. M. Mangini, G. Pedroncelli, and W. Ukovich, "Assignment of electrical vehicles to charging stations by a distributed approach," in *Proc. Eur. Control Conf. (ECC)*, Jun. 2014, pp. 1888–1893.

[12] H. Nabati and J. Y. Yu, "Distributed, private, and derandomized allocation algorithm for EV charging," in *Proc. IEEE Int. Smart Cities Conf. (ISC)*, Sep. 2018, pp. 1–8.

[13] L. Ni, B. Sun, X. Tan, and D. H. K. Tsang, "Online price-based vehicle-to-station recommendations for EV battery swapping," in *Proc. IEEE Int. Conf. Commun., Control, Comput. Technol. Smart Grids (SmartGridComm)*, Oct. 2018, pp. 1–6.

[14] Y. Nie, C. Y. Chung, L. Chen, X. Wang, R. Yu, and M. Qin, "Inventory management of DSCC system via improved (s, S) model," in *Proc. IEEE PES Gen. Meeting Conf. Expo.*, Jul. 2014, pp. 1–5.

[15] B. Sun, X. Sun, D. H. K. Tsang, and W. Whitt, "Optimal battery purchasing and charging strategy at electric vehicle battery swap stations," *Eur. J. Oper. Res.*, vol. 279, no. 2, pp. 524–539, 2019.

[16] H. Xu, D. D. Yao, and S. Zheng, "Optimal control of replenishment and substitution in an inventory system with nonstationary batch demand," *Prod. Oper. Manage.*, vol. 20, no. 5, pp. 727–736, Sep. 2011.

[17] D. D. Yao, S. X. Zhou, and W. Zhuang, "Joint initial stocking and transshipment-asymptotics and bounds," *Prod. Oper. Manage.*, vol. 25, no. 2, pp. 273–289, Feb. 2016.

[18] S. Faridimehr, S. Venkatachalam, and R. B. Chinnam, "A stochastic programming approach for electric vehicle charging network design," *IEEE Trans. Intell. Transp. Syst.*, vol. 20, no. 5, pp. 1870–1882, May 2019.

[19] F. Wu and R. Sioshansi, "A two-stage stochastic optimization model for scheduling electric vehicle charging loads to relieve distribution-system constraints," *Transp. Res. B, Methodol.*, vol. 102, pp. 55–82, Aug. 2017.

[20] T. Guo, P. You, and Z. Yang, "Recommendation of geographic distributed charging stations for electric vehicles: A game theoretical approach," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, Jul. 2017, pp. 1–5.

[21] Z. Tian *et al.*, "Real-time charging station recommendation system for electric-vehicle taxis," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 11, pp. 3098–3109, Nov. 2016.

[22] H. Topaloglu, "Joint stocking and product offer decisions under the multinomial logit model," *Prod. Oper. Manage.*, vol. 22, no. 5, pp. 1182–1199, 2013.

[23] G. Gallego, R. Ratliff, and S. Shebalov, "A general attraction model and sales-based linear program for network revenue management under customer choice," *Oper. Res.*, vol. 63, no. 1, pp. 212–232, Feb. 2015.

[24] G. Gallego, A. Li, V.-A. Truong, and X. Wang, "Online resource allocation with customer choice," 2015, *arXiv:1511.01837*. [Online]. Available: http://arxiv.org/abs/1511.01837

[25] W. Ma and D. Simchi-Levi, "Algorithms for online matching, assortment, and pricing with tight weight-dependent competitive ratios," *Oper. Res.*, Jun. 2017.

[26] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. Cham, Switzerland: Springer, 2011.

[27] L. Ni, B. Sun, X. Tan, and D. H. K. Tsang. *Online Appendix*. Accessed: Jun. 2020. [Online]. Available: https://www.dropbox.com/s/m7t8derjy6uxpck/Appendix_Section.pdf?dl=0

[28] A. J. Kleywegt, A. Shapiro, and T. Homem-de-Mello, "The sample average approximation method for stochastic discrete optimization," *SIAM J. Optim.*, vol. 12, no. 2, pp. 479–502, Jan. 2002.

[29] N. Buchbinder, K. Jain, and J. S. Naor, "Online primal-dual algorithms for maximizing ad-auctions revenue," in *Proc. Eur. Symp. Algorithms*. Cham, Switzerland: Springer, 2007, pp. 253–264.

[30] A. Borodin and R. El-Yaniv, *Online Computation and Competitive Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

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